This lab implements MATLAB code relative to the concept of change of basis in $\mathbb{R}^n$.

**What you have to submit:** Submit the required MATLAB code from TASK I electronically by e-mail to your lab instructor. Name the file `YourLastnameFirstnameI.m`.

**Important notice:** Lab assignments must be submitted using your KSU email address. Lab assignments submitted from a non-KSU email address will not be considered. Files with an incorrect extension will not be considered. Please write in the subject of your email: Lab 7 and your first name and last name.

**TASK I.**

**Warm up.** Consider the following ordered bases for $\mathbb{R}^3$:

- $E = \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix} \right\}$
- $F = \{ \vec{u}_1, \vec{u}_2, \vec{u}_3 \} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

(a) Find the transition matrix $T$ from the basis $E$ to the basis $F$. This is, $T$ is a $3 \times 3$ matrix such that if $[\vec{w}]_E$ denotes the coordinates of a vector $\vec{w}$ in $\mathbb{R}^3$ with respect to the basis $E$ and $[\vec{w}]_F$ denotes the coordinates of $\vec{w}$ with respect to the basis $F$, then

$$[\vec{w}]_F = T [\vec{w}]_E.$$

(b) If $\vec{w} = 3\vec{v}_1 + 2\vec{v}_2 - \vec{v}_3$ find the coordinates of $\vec{w}$ with respect to the basis $F$.

**Solution:**

(a) Consider the matrices $T_E$ and $T_F$ whose columns are given, respectively, by the ordered vectors in $E$ and $F$,

$$T_E = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 5 \\ 1 & 2 & 4 \end{bmatrix} \quad T_F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$
Recall from lecture that the transition matrix $T$ from $E$ to $F$ is given by $T_F^{-1}T_E$, this is

\[ T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 5 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 5 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -3 \\ -1 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix} \]

(b) Using the transition matrix found in (a) and that the coordinates of $\vec{w}$ with respect to the basis $E$ are 3, 2, and $-1$, respectively,

\[ T[\vec{w}]_E = \begin{bmatrix} 1 & 1 & -3 \\ -1 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 3 \end{bmatrix} = [\vec{w}]_F. \]

Then $\vec{w} = 8\vec{u}_1 - 5\vec{u}_2 + 3\vec{u}_3$.

**What you have to do.** Write an M-file function that has as inputs two matrices of size $n \times n$ and a column vector of size $n \times 1$. The columns of each of the matrices constitute ordered bases, $E$ and $F$, for $\mathbb{R}^n$ and the components of the column vector are the coordinates of a vector $\vec{w}$ of $\mathbb{R}^n$ with respect to the basis $E$ (compare with the warm up). The output of the function is the transition matrix $T$ from basis $E$ to basis $F$, and a column vector of size $n \times 1$ whose components are the coordinates of $\vec{w}$ with respect to the basis $F$. Your code should produce an error message if the dimensions of the entered matrices and/or vector do not agree or if the columns of one of the entered matrices do not constitute a basis for $\mathbb{R}^n$.

Try the pseudo code `lab7taskI.p` to check how your program should work. Type

```matlab
>> [T,v]=lab7taskI(TE,TF,wE)
```

where $TE$ and $TF$ are matrices whose columns are the ordered bases, $E$ and $F$, respectively, given in the appendix, and $wE$ is the column vector whose components are the coordinates of the vector $\vec{w}$ with respect to the basis $E$, given in the appendix.

**APPENDIX.**

(a) $E$, $F$ and $\vec{w}$ as in the warm up of TASK I.

(b) $E = \{\vec{v}_1, \vec{v}_2\} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$, $F = \{\vec{u}_1, \vec{u}_2\} = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \end{bmatrix} \right\}$,

$\vec{w} = 4\vec{v}_1 - 5\vec{v}_2$.

(c) $E = \{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\}$ is the standard basis in $\mathbb{R}^4$, $F = \{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ where

\[
\begin{align*}
\vec{u}_1 &= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, & \vec{u}_2 &= \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, & \vec{u}_3 &= \begin{bmatrix} -2 \\ -1 \\ 2 \\ 3 \end{bmatrix}, & \vec{u}_4 &= \begin{bmatrix} 1 \\ -1 \\ -2 \\ 0 \end{bmatrix},
\end{align*}
\]
and \( \vec{w} = -\vec{e}_1 + 7\vec{e}_2 + 8\vec{e}_3 - 2\vec{e}_4. \)

(d) \( E = \{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\} \) as in part (c) and \( F = \{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\} \), the standard basis in \( \mathbb{R}^4 \), and \( \vec{w} = \vec{u}_1. \)

To check for error messages use:

(a) Dimensions do not agree:

\[
E = \{\vec{v}_1, \vec{v}_2\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} \right\}, \quad F = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}, \quad \vec{w} = 8\vec{v}_1 - 7\vec{v}_2.
\]

(b) One of the sets is not a basis for \( \mathbb{R}^n \):

\[
E = \{\vec{v}_1, \vec{v}_2\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}, \quad F = \{\vec{u}_1, \vec{u}_2\} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}, \quad \vec{w} = 4\vec{v}_1 - 5\vec{v}_2.
\]