The point value of each problem is given in the margin. Standard notation is used:

- $(a, b) = \text{GCD}$,
- $[a, b] = \text{LCM}$,
- $p^k | n$ if $p^k | n$ but $p^{k+1} \nmid n$.
- $d(n) = \text{the number of positive divisors of } n$.
- $\sigma(n) = \text{the sum of the positive divisors on } n$.
- $\phi(n) = \text{the number of integers relatively prime to } n \text{ from } 1 \text{ to } n$.
- $\mu(n) = 1$ if $n = 1$, $0$ if $p^2 | n$, and $(-1)^k$ if $n = p_1 \ldots p_k$.

1. Let $a = 2^5 5^8 7^2$, $b = 2^9 3^5 5^7$. Find the following.
   (i) $(a, b) =$ The prime factorization will do!
   (ii) $[a, b] =$ Same comment.
   (iii) The value $e$ such that $2^e | b^2 a$
   (iv) The value $f$ such that $5^f | (b - a)$.

2. Use the Sieve of Eratosthenes to find all the primes between 180 and 200.
   What is the largest prime divisor that must be sifted out?

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   Primes:

3. (a) If $2^k + 1$ is a prime, what can be said about $k$?
   (b) Find a nontrivial factor of $10^9 + 1$. (It doesn’t have to be a prime.)
   (c) Give a prime divisor of $2^{45} - 1$. (One will do.)
4. Find the following.
   (a) The prime power factorization of 270.
   (b) \( d(270) = \)
   (c) \( \sigma(270) = \)
   (d) \( \phi(270) = \)
   (e) Is 270 abundant, deficient or perfect?

5. Suppose that \( f \) is a multiplicative function defined on \( N \) such that \( f(2) = 3, f(3) = 5, f(5) = 10 \) and \( f(p^2) = 0 \) for any prime \( p \). For each of the following find the value or state that it cannot be determined based on the given information.
   (a) \( f(30) = \)
   (b) \( f(50) = \)
   (c) \( f(27) = \)
(12) 6. Give a proof that there are infinitely many primes.

(10) 7. Let \( n = 2^k(2^{k+1} - 1) \), where \( k \) is a positive integer and \( 2^{k+1} - 1 \) is a prime. Prove that \( n \) is a perfect number, that is, \( \sigma(n) = 2n \).

(10) 8. Let \( F(n) = \sum_{d\mid n} d\mu(d) \).
   
   (a) Show that the function \( f(n) = n\mu(n) \) is multiplicative. (You may assume \( \mu \) is multiplicative.)

   (b) Find \( F(p^k) \) for any prime power \( p^k \).

   (c) Find \( F(3000) = \)