INTRODUCTION TO NUMBER THEORY
Exam 1
February 18, 2000

The point value of each problem is given in the margin.

(10) 1. Find integers $q, r$ such that $-25 = 6q + r$ with
   a) $0 \leq r < 6$.

   b) $|r| \leq 3$.

(10) 2. Use the Euclidean Algorithm to find the greatest common divisor of 42 and 280.

(14) 3. Find the general solution of the linear equation $46x - 28y = 6$. 
4. Use properties of congruences to compute the least residue of the following numbers modulo 7. (Avoid long multiplication in $\mathbb{Z}$.)

(a) $707 \cdot 145 - 17$

(b) $78^2 + 72^5$

5. Prove the following theorem. If $a, b, c$ are integers such that $a|bc$ and $(a, b) = 1$ then $a|c$.

6. True, False. Circle T or F. True means that the statement is true for all choices of integers $a, b, c, d$. $(a, b) = \text{GCD.} [a, b] = \text{LCM}$.

- T F a) For any integer $a$, $0|a$.
- T F b) If $a|b$ and $a|c$ then $a|(2b - c)$.
- T F c) If $a|bc$ then either $a|b$ or $a|c$.
- T F d) If $d|a$ and $d|b$ then $d|[a, b]$.
- T F e) For any integer $q$, $(17 - 5q, 5) = 1$
- T F f) If $6|(a + b)$ then $a \equiv -b \pmod{6}$.
- T F g) For any $a$, the equation $3x - 6y = a$ has a solution in $\mathbb{Z}$.
- T F h) Any integer can be expressed as a linear combination of 7 and 11.
- T F i) If $f_1 = 1, f_2 = 1, f_3 = 2, ...$ is the Fibonacci sequence, then for any positive integer $n$, $f_n + f_{2n} = f_{3n}$.
(12) 7. Prove by induction that $4^n \equiv 1 + 3n \pmod{9}$, for any positive integer $n$.

(12) 8. Prove that any positive integer $n > 1$ can be expressed as a product of primes.