1. (12 points) Let \( a = 2^3 5^8 13^2, b = 2^5 3^7 5^3 \). Find the following:

(a) The prime factorization of \((a, b)\)

(b) The prime factorization of \([a, b]\)

(c) The value of \(e\) if \(2^e || a^3 b\)

(d) The value of \(f\) if \(5^f || (a + b)\)

2. (10 points) Use induction to prove that for all positive integers \(n\)

\[ 11^n \equiv 10n + 1 \pmod{100}. \]

3. (8 points) Sieve out the primes from 148 to 165. Primes =

\[ 148 \quad 149 \quad 150 \quad 151 \quad 152 \quad 153 \quad 154 \quad 155 \quad 156 \quad 157 \quad 158 \quad 159 \quad 160 \quad 161 \quad 162 \quad 163 \quad 164 \quad 165 \]

Which prime divisors had to be sifted out?
4. (22 points) Let $p$, $q$ denote distinct primes, and recall that

$$\tau(n) = \text{the number of positive divisors of } n, \quad \sigma(n) = \text{the sum of the positive divisors of } n.$$ 

(a) Find the prime factorization of $350 = \ldots$

(b) $\tau(350) = \ldots$

(c) $\sigma(350) = \ldots$

(d) $\tau(p^4q^2) = \ldots$

(e) Give the positive divisors of $p^4q^2$ (table form is fine).

(f) Describe the prime factorizations possible for $n$ if $\tau(n) = 10$.

(g) What is the smallest positive integer $n$ with $\tau(n) = 10$?

5. (10 points) Prove that any postal amount greater than or equal to 12 cents can be made up using just 3 cent and 7 cent stamps.
6. (8 points)
(a) Find the missing digit ? that makes $1260054?413000$ a multiple of 9.

(b) Find $h$ if $2^h || 130517986325191173251852$.

7. (20 points) Circle True (T) or False (F).
   
   T F (a) If $p$ is a prime then $p|a^3b^2$ implies that $p|a$ or $p|b$.
   
   T F (b) If $x$ is rational and $y$ is irrational then $(x + y)$ is irrational.
   
   T F (c) If a positive integer $n < 280$ has no divisor $1 < d \leq 14$ then $n$ is prime.
   
   T F (d) The number of primes from 1 to $x$, denoted $\pi(x)$, satisfies $\lim_{x \to \infty} \frac{\pi(x)}{x \ln x} = 1$.
   
   T F (e) If $f(n)$ is multiplicative then $f(18) = f(3)f(6)$.
   
   T F (f) If $p|a$ and $p|b$ then $p||(a + b)$.
   
   T F (g) The Fibonacci numbers satisfy $f_{3n} = f_{2n} + f_n$.
   
   T F (h) The least residue modulo 11 of $\overbrace{33333333333333}^{16 \text{ times}}$ is 3.
   
   T F (i) The Fibonacci numbers satisfy $f_{n+2} = 2f_n + f_{n-1}$.
   
   T F (j) If $a = 2^35^213$ and $b = 2^{11}5^3$ then $b|a^4$.

8. (10 points) Prove that $\sqrt{18}$ is irrational.