1. (12 points) Consider the weighted voting system $[17 : 8, 7, 3, 2, 1]$ with five players $P_1, P_2, P_3, P_4, P_5$.

(a) List any dictators _________. List the players with veto power: _________.

(b) Write down the winning coalitions and circle any players that are critical.

(c) Find the Banzhaf power distribution: $\beta_1 = ____$, $\beta_2 = ____$, $\beta_3 = ____$, $\beta_4 = ____$, $\beta_5 = ____$.

2. (10 points) Consider the weighted voting system $[9 : 6, 5, 3]$ with three players $P_1, P_2, P_3$.

(a) Write down the sequential coalitions and circle the pivotal player.

(b) Find the Shapley-Shubik power distribution: $\sigma_1 = ______$, $\sigma_2 = ______$, $\sigma_3 = ______$.

3. (8 points) Suppose that a weighted voting system has 10 players.

(a) The total number of coalitions (subsets) with at least two players is ________.

(b) The number of coalitions with 4 players is ________.
4. (10 points) Anne and Bob want to divide a pizza worth $12 using the divider choose method. Half of the pizza is mushroom and half pepperoni. The table shows how much they each think the halves are worth. Anne loses the coin toss and has to be the divider.

<table>
<thead>
<tr>
<th></th>
<th>Mushroom</th>
<th>Pepperoni</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anne</td>
<td>$4</td>
<td>$8</td>
</tr>
<tr>
<td>Bob</td>
<td>$9</td>
<td>$3</td>
</tr>
</tbody>
</table>

(a) The solid line is Anne’s first cut. At what angle to the dotted line should she make her second cut?

(b) Does Bob pick the left or right piece? __________. To Bob the value of his piece is $__________.

5. (10 points) Abby, Bill, Chloe, Dan and Emily share a cake using the lone-divider method. Abby cuts the cake into five parts \( s_1, s_2, s_3, s_4, s_5 \) that she considers equal. The table shows the value each places on the pieces.

<table>
<thead>
<tr>
<th></th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abby</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Bill</td>
<td>10%</td>
<td>10%</td>
<td>40%</td>
<td>10%</td>
<td>30%</td>
</tr>
<tr>
<td>Chloe</td>
<td>19%</td>
<td>19%</td>
<td>30%</td>
<td>17%</td>
<td>15%</td>
</tr>
<tr>
<td>Dan</td>
<td>40%</td>
<td>30%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Emily</td>
<td>50%</td>
<td>10%</td>
<td>15%</td>
<td>10%</td>
<td>15%</td>
</tr>
</tbody>
</table>

(a) Determine the bids:
Bill __________, Chloe __________, Dan __________, Emily __________.

(b) Determine who gets which piece:
Abby __________, Bill __________, Chloe __________, Dan __________, Emily __________.

6. (14 points) Two dice are rolled.

(a) There are ________ outcomes are in the sample space.

(b) List the missing outcomes in the event ‘the dice totalled at least 10’:
\[ E = \{(4, 6), (6, 4), \} \]

(c) The probability of rolling at least 10 is \( P(E) = \) ________.

(d) The odds in favor of rolling at least 10 are ________.
7. (14 points) (a) Amy, Beth, Caleb and Duncan decide to sort their halloween candy by type then share it out using the method of sealed bids. Their dollar valuations are as shown. Fill in the fair-share values.

<table>
<thead>
<tr>
<th></th>
<th>Amy</th>
<th>Beth</th>
<th>Caleb</th>
<th>Duncan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reeses</td>
<td>10</td>
<td>6</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Snickers</td>
<td>15</td>
<td>16</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>MilkyWays</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Starbursts</td>
<td>4</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>


(c) How much does each person now owe to the estate (positive) or need to receive (negative) from it:
Amy ________, Beth ________, Caleb ________, Duncan ________.

(d) Find the surplus after the first settlement ________.

(e) Find the final settlement (which property they get and amount of cash they pay or receive (in thousands)
Amy: Property: ____________, Pay: ________, Receive: ________.
Beth: Property: ____________, Pay: ________, Receive: ________.
Caleb: Property: ____________, Pay: ________, Receive: ________.
Duncan: Property: ____________, Pay: ________, Receive: ________.

8. (10 points) Find the mean, median, standard deviation, and 40th percentile for the 6 exam scores:
90, 80, 75, 73, 85, 83.

Mean: $\mu = ______$. Median: $M = ______$. Standard Deviation: $\sigma = ______$. $X_{40} = ______$.

9. (10 points) Run Kruksal’s Algorithm to find a minimum spanning tree for the graph.
Total cost/weight: ________.
10. (20 points) The table shows the preference schedule for an election with 5 candidates and 25 voters.

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>8</th>
<th>7</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>C</td>
<td>B</td>
<td>D</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>2nd</td>
<td>E</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>3rd</td>
<td>D</td>
<td>B</td>
<td>D</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>4th</td>
<td>B</td>
<td>A</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>B</td>
</tr>
<tr>
<td>5th</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>B</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

(a) Use the **Borda count method** method to rank the candidates.

Ranking: ________________________.

(b) Use the **Plurality with elimination** to rank the candidates.

Ranking: ________________________.

11. (6 points) A large jar contains 500 red and green gumballs (well mixed). Britanny draws out a sample of gumballs. It contains 8 red and 32 green. Estimate the number \( R \) of red gumballs in the jar.
12. (20 points) In this project digraph the task times are given in hours.

\[
\begin{array}{c}
\text{START}(0) \rightarrow A(2) \rightarrow D(3) \rightarrow F(4) \rightarrow \text{END}(0) \\
\text{START}(0) \rightarrow B(5) \rightarrow D(3) \rightarrow G(7) \rightarrow \text{END}(0) \\
\text{START}(0) \rightarrow C(6) \rightarrow E(8) \rightarrow \text{END}(0)
\end{array}
\]

a. Use the decreasing-time algorithm to schedule the project using 3 processors.

Decreasing-time priority list: ____________________________________________________________________.
Project finishing time = ____ hours

\[
\begin{array}{cccccccc}
0 & 5 & 10 & 15 & 20 & 25 & 30 & \text{Time} \\
P_1 & & & & & & & \\
P_2 & & & & & & & \\
P_3 & & & & & & & \\
\end{array}
\]

b. Run the backflow algorithm (write the critical time next to each vertex).
The length of the critical path for the project is ____ hours.

c. Use the critical-path algorithm to schedule the project using 2 processors.

Critical-time priority list: ____________________________________________________________________.
Project finishing time = ____ hours

\[
\begin{array}{cccccccc}
0 & 5 & 10 & 15 & 20 & 25 & 30 & \text{Time} \\
P_1 & & & & & & & \\
P_2 & & & & & & & \\
\end{array}
\]
13. (8 points) A security guard needs to patrol the streets shown in the graph.

(a) Starting and finishing in the same place is it possible to patrol without going over some streets twice?
(b) If he can start and finish in different places how many streets does he have to walk over twice?
Mark the start & finish and the street(s) involved.

14. (16 points) Hannah is delivering Christmas presents to the houses of 5 relatives A, B, C, D and E, starting and finishing at her parents house A. The distance in miles between the houses are shown. Hannah decides to use two algorithms that she saw in MATH160 to plan her route.

(a) Run the nearest neighbor algorithm starting at A.

(b) Now run the cheapest link algorithm.

The tour from this algorithm is A, __________, A, and is ___ miles long.

The tour from this algorithm is A, __________, A, and is ___ miles long.