You are welcome to work together but everyone needs to write up distinct solutions. If you use any books or other people heavily, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to \LaTeX.

For problems 1-4 assume \( H \) and \( K \) are groups, \( \phi \) a homomorphism from \( K \) into \( \text{Aut}(H) \), and identify \( H \) and \( K \) as subgroups of \( G = H \rtimes_\phi K \).

1. Section 5.5 \#1. Prove that \( C_K(H) = \ker \phi \). [Hint: \( C_K(H) = C_G(H) \cap K \)].

2. Section 5.5 \#16. Show that there are exactly 4 distinct homomorphisms from \( Z_2 \) into \( \text{Aut}(Z_8) \). Prove that two of the resulting semidirect products are isomorphic to \( Z_8 \times Z_2 \) and \( D_{16} \).

3. Section 5.5 \#18. Show that if \( H \) is any group then there is a group \( G \) that contains \( H \) as a normal subgroup with the property that for every automorphism \( \sigma \) for \( H \) there is an element \( g \in G \) such that conjugation by \( G \) when restricted to \( H \) is the given automorphism \( \sigma \).

4. Section 5.5 \#21. Let \( p \) be an odd prime and let \( P \) be a \( p \)-group. Prove that if every subgroup of \( P \) is normal then \( P \) is abelian. [Hint: You may find Exercise 20 of section 5.5 useful.]

5. Section 6.2 \#14. Prove there are no simple groups of order 144.

6. Section 6.3 \#2. Prove that if \( |S| > 1 \) then \( F(S) \) is non-abelian.

7. Section 6.3 \#4. Prove that every nonidentity element of a free group is of infinite order.

8. Exhibit all degree 1 complex representations of a finite abelian group. Deduce that the number of such representations equals the order of the group. [Hint: First decompose the abelian group into a direct product of cyclic groups.]

9. Let \( X \) be a finite set on which \( G \) acts and let \( \rho \) be the corresponding permutation representation and let \( \chi_X \) be the character of \( \rho \). Let \( g \in G \) and show that \( \chi_X(g) \) is the number of elements of \( X \) fixed by \( g \).