You are welcome to work together but everyone needs to write up distinct solutions. If you use any books or other people heavily, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrociou s, I am happy to give you a basic introduction to \LaTeX.

1. Section 1.1 #20. For \( x \) an element in \( G \) show that \( x \) and \( x^{-1} \) have the same order.

2. Section 1.1 #24. If \( a \) and \( b \) are commuting elements of \( G \), prove that \((ab)^n = a^n b^n\) for all \( n \in \mathbb{Z} \). [Hint: Induction on positive \( n \) first.]

3. Section 1.1 #25. Prove that if \( x^2 = 1 \) for all \( x \in G \) then \( G \) is abelian.

4. Section 1.3 #10. Prove that if \( \sigma \) is the \( m \)-cycle \((a_1 \ a_2 \ldots \ a_m)\), then for all \( i \in \{1,2,\ldots,m\} \), \( \sigma^i(a_k) = a_{k+i} \) where \( k+i \) is replaced by its least positive residue mod \( m \). Deduce that \( |\sigma| = m \).

5. Section 1.3 #15. Prove that the order of an element in \( S_n \) equals the least common multiple of the lengths of cycles in its cycle decomposition. [Hint: Use 1.1 24 and 1.3 10].

6. Find the maximum possible order of an element in \( S_{12} \) and exhibit an element of that order.

7. Section 1.6 #9. Prove that \( D_{24} \) anad \( S_4 \) are not isomorphic.

8. Section 1.7 #16. Let \( G \) be any group and let \( A = G \). Show that the maps defined by \( g \cdot a = gag^{-1} \) for all \( g \cdot a \in G \) satisfy the axioms of a (left) group action of \( G \) on itself.