Solution for section 6.2, B8a.

\[
\left(\frac{2}{5} + \frac{5}{8}\right) + \frac{3}{5} = \left(\frac{2}{5} + \frac{3}{5}\right) + \frac{5}{8} = 1 + \frac{5}{8} = \frac{13}{8}.
\]

Solution for section 6.2, B9a. Using equations, adding \(\frac{2}{9}\) to both numbers, we get

\[
\frac{2}{9} - \frac{7}{9} = \frac{4}{9} - \frac{29}{9} = \frac{5}{9} - 3 = \frac{24}{9}.
\]


Using the definition (cross-multiplying) we have

\[
\frac{2}{5} < \frac{5}{8}, \text{ since } 8 \cdot 2 < 5 \cdot 5.
\]

Finding a common denominator we have

\[
\frac{2}{5} < \frac{5}{8}, \text{ since } \frac{16}{40} < \frac{25}{40}.
\]

Solution for section 6.2, B19. The first thing we should do is find the total amount of the time Kathleen devoted to studying. We know each portion of time studied by subject except for English. So we have to find the fraction of time Kathleen spent studying for English. We do this by subtracting the time devoted to the other subjects from the total time. Note that total amount of the study time in terms of a fraction is 1, since the total study time is the "whole." Doing the computation:

\[
1 - \frac{2}{5} - \frac{3}{20} - \frac{1}{3} = \frac{60}{60} - \frac{24}{60} - \frac{20}{60} - \frac{7}{60}.
\]

Therefore, Kathleen spent \(\frac{7}{60}\) of total study time on English. To find the total study time we have to ask ourselves the question for what total time is 7 out of 60 equivalent to 35 minutes. One way to answer this question is to set up a proportion, that is,

\[
\frac{\text{part}}{\text{whole}} = \frac{7}{60} = \frac{35}{T},
\]

where \(T\) is the total time devoted to studying. Solving for \(T\) we find that \(T = 300\) minutes.

To find the time spent studying Spanish we take \(\frac{3}{20}\) of 300 minutes, that is,

\[
\frac{3}{20} \cdot 300 = 45 \text{ minutes},
\]

which is the answer.

Solution for section 6.2, B22.

(a) \(8^2 + 15^2 = 64 + 225 = 289 = 17^2\).

(b)

i. \(\frac{1}{7} + \frac{1}{9} = \frac{16}{63}, 16^2 + 63^2 = 65^2\).

ii. \(\frac{1}{11} + \frac{1}{13} = \frac{24}{143}, 24^2 + 143^2 = 145^2\).

iii. \(\frac{19}{21} + \frac{9}{7} = \frac{399}{399}, 40^2 + 399^2 = 401^2\).

(c) The third number \(c\), in \(a^2 + b^2 = c^2\), is two more than the denominator of the sum.

(d)

\[
\frac{1}{n} + \frac{1}{n+2} = \frac{2n + 2}{n^2 + 2n}, \quad (2n + 2)^2 + (n^2 + 2n)^2 = (n^2 + 2n + 2)^2.
\]

Date: Spring 2004.

(a) \[
\begin{align*}
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{7}{8} = \frac{2^3 - 1}{2^3} \\
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &= \frac{15}{16} = \frac{2^4 - 1}{2^4} \\
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} &= \frac{31}{32} = \frac{2^5 - 1}{2^5}
\end{align*}
\]

(b) Taking into the account the pattern of the sum of the terms found in part (a), we find the answer to this question by finding the smallest power of two that exceeds or is equal to 100. This smallest power of two is 7 since \(2^6 = 64\) and \(2^7 = 128\). Then the sum of the first seven terms of this sequence is \(\frac{127}{128} \geq \frac{99}{100}\).

(c) Notice the pattern of the finite sums of this sequence, that is,

<table>
<thead>
<tr>
<th>Terms</th>
<th>Sums</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 terms</td>
<td>(\frac{7}{8})</td>
</tr>
<tr>
<td>4 terms</td>
<td>(\frac{15}{16})</td>
</tr>
<tr>
<td>5 terms</td>
<td>(\frac{31}{32})</td>
</tr>
<tr>
<td>6 terms</td>
<td>(\frac{63}{64})</td>
</tr>
<tr>
<td>7 terms</td>
<td>(\frac{127}{128})</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Since each of the sums is less than 1 but keeping getting closer to 1, the best guess would be that the sum of this geometric series is 1.

Solution for section 6.3, B1c. The area model for the product \(\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}\) is depicted as


(a) \(\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}\)

(b) \(\frac{12}{7} \times \frac{4}{9} = \frac{48}{63}\)

Solution for section 6.3, B4.

(a) \[
\begin{align*}
\frac{3}{16}, \frac{5}{8}, \frac{9}{10}, \frac{7}{5}.
\end{align*}
\]

(b) \[
\begin{align*}
\frac{5}{7}, \frac{10}{9}, \frac{8}{5}, \frac{16}{3}.
\end{align*}
\]

(c) The orders are reversed.

\[
\frac{2}{7} < \frac{2 + 3}{7 + 8} < \frac{3}{8} \quad \text{and} \quad \frac{2}{7} = \frac{16}{56} < \frac{18}{56} < \frac{21}{56} = \frac{3}{8}
\]

Solution for section 6.3, B20. Let \( b \) represent the amount of bottles. Then the number of cans is 54 more than this, or \( b + 54 \). Then the total number of beverage containers is \( b + b + 54 = 2b + 54 \). Since the number of bottles is \( \frac{5}{11} \) of the total number of beverage containers, we can say

\[
\frac{b}{2b + 54} = \frac{5}{11}.
\]

Cross-multiplying we get

\[
11b = 5(2b + 54).
\]

Solving for \( b \) we find that \( b = 270 \), which is the number of bottles collected.

Solution for section 6.3, B26. In this problem \( \frac{2}{3} \) is the “whole” and the \( \frac{1}{2} \) is “part” and want to how much the “part” is of the “whole”. This corresponds to division, hence the answer is

\[
\frac{1}{2} \div \frac{2}{3} = \frac{1 \cdot 3}{2 \cdot 2} = \frac{3}{4}.
\]

This means that \( \frac{1}{3} \) is \( \frac{3}{4} \) of \( \frac{2}{3} \). Therefore we can make \( \frac{3}{7} \) of the recipe.

Solution for section 6.3, B27.

Abigail: \( \frac{4}{12} = \frac{1}{3} \) Harold: \( \frac{24}{20} = \frac{6}{5} \)

Solution for section 7.1, B5ab.

(a) Since the denominator of

\[
\frac{2^4 \cdot 11^{19} \cdot 17^{19}}{5^{12}},
\]

has only a power of 5 as a divisor, this fraction has a terminating decimal representation.

(b) First we must reduce this fraction, that is,

\[
\frac{2^3 \cdot 3^{11} \cdot 7^{9} \cdot 11^{16}}{7^{13} \cdot 11^9 \cdot 5^7} = \frac{2^3 \cdot 3^{11} \cdot 11^7}{7^4 \cdot 5^7}.
\]

Since this fraction has a number, 7, that is not a 2 or 5 as a divisor, this fraction does not have a terminating decimal representation.

Solution for section 7.1, B7b.

\[
\frac{4 \quad 2 \quad 3}{11 \cdot 5^7}
\]

Solution for section 7.1, B11be.

(b) \( 4.52 \times 10^3 \)

(c) \( 1.084 \times 10^9 \)

Solution for section 7.1, B12bc.

(b)

\[
\frac{7.8752}{10,000,000} = 7.8752 \times 10^{-7}.
\]

(c)

\[
(8.24 \times 10^{30})(10^7) = 8.24 \times 10^{37}.
\]

Solution for section 7.1, B16ab.

(a) \( 123.9 \div 5.3 \approx 125 \div 5 = 5. \)

(b) \( 87.4 \times 7.9 \approx 90 \times 8 = 720. \)

Solution for section 7.1, B17.

(a) 321.09

(b) 12.162

(c) 4.009

(d) 2.0

(e) 2.00

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