Solution for section 5.1, B1. Multiples of 2 in columns 2, 4, and 6; multiples of 3 in columns 3 and 6; multiples of diagonals moving down from right to left; multiples of 7 on the diagonals moving down from left to right.

Solution for section 5.1, B4gi.

(g) First notice that we should write 6000 in terms of its prime factors, that is, \( 6000 = 2^4 \cdot 3 \cdot 5^3 \). Then

\[
\begin{align*}
\frac{6000}{(2^{21} \times 3^{17} \times 5^{39} \times 29^{37})} & = \frac{(2^4 \times 3 \times 5^3)}{(2^{21} \times 3^{17} \times 5^{39} \times 29^{37})}.
\end{align*}
\]

Now, the above statement is true because

\[
\begin{align*}
(2^4 \times 3 \times 5^3) & \times (2^{17} \times 3^{16} \times 5^{86} \times 29^{37}) = (2^{21} \times 3^{17} \times 5^{39} \times 29^{37}).
\end{align*}
\]

(i) As in part (g), this is true because

\[
\begin{align*}
(p^3 q^3 r) \cdot (p^2 q^6 r^6 s^2 t^{27}) = p^5 q^9 r^7 s^2 t^{27}.
\end{align*}
\]

Solution for section 5.1, B14. Assume \( 9 \mid (a + b + c + d) \)

Let \( r = abcd = a \cdot 10^3 + b \cdot 10^2 + c \cdot 10^1 + d \)

represent any four digit number. Then we rewrite \( r \) as

\[
\begin{align*}
r & = a \cdot 10^3 + b \cdot 10^2 + c \cdot 10^1 + d \\
& = a(999 + 1) + b(99 + 1) + c(9 + 1) + d \\
& = 999a + 99b + 9c + (a + b + c + d) \\
& = 9(111a + 11b + c) + (a + b + c + d).
\end{align*}
\]

Now, since \( 9 \mid (a + b + c + d) \) and \( 9 \mid (9(111a + 11b + c)) \), then 9 must divide

\[
r = 9(111a + 11b + c) + (a + b + c + d),
\]

which completes the proof.

Solution for section 5.1, B34. This is true and can be proved using the divisibility test by 3.

Solution for section 5.2, B3. The \( \text{LCM}(12, 18) \) is larger. Keep in mind that the largest the \( \text{GCF}(12, 18) \) could possibly be is 12 and the smallest the \( \text{LCM}(12, 18) \) could possibly be is 18.

Solution for section 5.2, B14bc. This is an application of the counting factors theorem on page 198.

(b) The only numbers that have exactly 3 divisors are numbers that are prime numbers squared, that is, \( p^2 \) where \( p \) is a prime.

(c) The are two types of numbers that have 4 divisors. Numbers of the form \( p \cdot q \), where \( p \) and \( q \) are prime, or \( p^3 \).

Solution for section 5.2, B16. The \( \text{GCF}(x^2, y^2) = 1 \). The prime factorizations of \( x \) and \( y \) share no prime factors because \( \text{GCF}(x, y) = 1 \). This means that numbers \( x^2 \) and \( y^2 \) also share no prime factors, since the same primes occur in \( x^2 \) and \( y^2 \) as in \( x \) and \( y \) but twice as often.

Solution for section 5.2, B23. The days the three cycles coincide are multiples of the \( \text{LCM}(23, 28, 33) = 21252 \). This means that the cycles occur in the same day every 21252 days.

Solution for section 6.1, B1.

(a) \( \frac{5}{8} \)  \hspace{1cm} (b) \( \frac{3}{10} \)  \hspace{1cm} (c) \( \frac{4}{6} \)  \hspace{1cm} (d) \( \frac{9}{4} \)

\[ \text{Date: Spring 2004.} \]
Solution for section 6.1, B2.
(a) One example of many possible answers {△△△△ □□□□}.
(b) 

(c) 

Solution for section 6.1, B4.
(a) False, since not every month has 30 days.
(b) True, since every week has 7 days.
(c) False, since not all months are the same size.

Solution for section 6.1, B13. Both Frank and Dave are right. Dave ate 3 more pieces than Frank. Now, 3 out of 12 is 1/4 while 3 out of 15 is 1/5. This means that when Dave said, ”I ate 1/4 more,” he meant out of 12 and when Frank said, “I ate 1/5 less,” he meant out of 15.

Solution for section 6.1, B17. There are infinitely many fractions less than \( \frac{1}{12} \). For example,
\[
\frac{1}{12} > \frac{1}{13} > \frac{1}{14} > \frac{1}{15} \ldots > 0,
\]
which implies that there is no “smallest” fraction greater than 0.

Solution for section 6.1, B18. The size of the unit different so a comparison cannot be made.

Solution for section 6.1, B20. A proper fraction means that the numerator is less than the denominator. Then if you add the same number to both the numerator and denominator to a proper fraction, you will get a number greater than your original fraction. For example,
\[
\frac{3}{5} < \frac{6}{8}.
\]