Solution for section 4.2, B11.
The change is $38 which is usually given by $20 + $10 + $5 + $1 + $1 + $1. Then returning the change we would count off 62, 63, 64, 65, 70, 80, 100.

Solution for section 4.2, B12.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<tbody>
<tr>
<td>a.</td>
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<td>b.</td>
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<td>1</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Solution for section 4.2, B18.

a. Halving Doubling

\[
\begin{align*}
35 \times 68 &= 2380 \\
17 \times 136 &= \\
A \times A &= \\
2 \times A &= \\
1 \times 2176 &= 
\end{align*}
\]

b. Halving Doubling

\[
\begin{align*}
\beta \beta \times \beta &= \\
19 \times 124 &= \\
9 \times 248 &= \\
\beta \times \beta &= \\
2 \times \beta &= \\
1 \times 1984 &= 
\end{align*}
\]

\[
124 + 248 + 1984 = 2356
\]

c. Halving Doubling

\[
\begin{align*}
31 \times 54 &= \\
15 \times 108 &= \\
7 \times 216 &= \\
3 \times 432 &= \\
1 \times 864 &= 
\end{align*}
\]

\[
54 + 108 + 216 + 432 + 864 = 1674
\]

Solution for section 4.2, B23.

Peter subtracted in the wrong order in the ones column and the tens column. Jeff did not regroup properly. The 10 in the tens place should be 9. John did not need to regroup a second time in the hundreds place.
Solution for section 4.3, B4a.
The lattice method works as follows:

\[
\begin{array}{c}
4 \\
+ 1 \\
\hline
6
\end{array}
\]
\[
\begin{array}{c}
7 \\
+ 3 \\
\hline
7
\end{array}
\]
\[
\begin{array}{c}
0 \\
+ 7 \\
\hline
5
\end{array}
\]
\[
\begin{array}{c}
1 \\
+ 2 \\
\hline
7
\end{array}
\]

Therefore the answer is 62seven.

Detail: 6seven + 3seven = 6seven + 1seven + 2seven = 10seven + 2seven = 12seven.

Solution for section 4.3, B5b.

\[
\begin{array}{c}
T \\
+ 9 \\
\hline
T
\end{array}
\]

Details:

T_{eleven} + 9_{eleven} = T_{eleven} + 1_{eleven} + 8_{eleven} = 10_{eleven} + 8_{eleven} = 18_{eleven}

Solution for section 4.3, B7bc.

b.

\[
\begin{array}{c}
10 \\
9 \\
\hline
E
\end{array}
\]

Details:

10_{twelve} + 9_{twelve} = 10_{twelve} - T_{twelve} + 9_{twelve} = 2_{twelve} + 9_{twelve} = E_{twelve}.

c.

\[
\begin{array}{c}
10 \\
7 \\
\hline
1
\end{array}
\]

Details: Keep in mind that 1_{eight} + 7_{eight} = 10_{eight}.

Solution for section 4.3, B9b.

Using the multiplication table we fill in the lattice to get an answer of 969_{twelve}.

<table>
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<tr>
<th>×</th>
<th>4</th>
<th>3_{twelve}</th>
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<tbody>
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<td>0</td>
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<tr>
<td>8</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>9_{twelve}</td>
</tr>
</tbody>
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<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>T</th>
<th>E</th>
</tr>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
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<td>T</td>
<td>E</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>T</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>1T</td>
</tr>
<tr>
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<td>3</td>
<td>6</td>
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<td>10</td>
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<td>4</td>
<td>8</td>
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<td>14</td>
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<td>20</td>
<td>24</td>
<td>28</td>
<td>30</td>
<td>34</td>
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<td>5</td>
<td>T</td>
<td>13</td>
<td>18</td>
<td>21</td>
<td>26</td>
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<td>39</td>
<td>42</td>
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<td>19</td>
<td>24</td>
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<td>20</td>
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<td>5T</td>
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<td>76</td>
<td>84</td>
<td>92</td>
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<tr>
<td>E</td>
<td>E</td>
<td>1T</td>
<td>29</td>
<td>38</td>
<td>47</td>
<td>56</td>
<td>65</td>
<td>74</td>
<td>83</td>
<td>92</td>
<td>T1</td>
</tr>
</tbody>
</table>

Base-twelve Multiplication table

Using the multiplication table we fill in the lattice to get an answer of 969_{twelve}.

Intermediate Algorithm

<table>
<thead>
<tr>
<th>×</th>
<th>4</th>
<th>3_{twelve}</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>9_{twelve}</td>
</tr>
</tbody>
</table>

Standard Algorithm

<table>
<thead>
<tr>
<th>×</th>
<th>4</th>
<th>3_{twelve}</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0_{twelve}</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>9_{twelve}</td>
</tr>
</tbody>
</table>
Solution for section 4.3, B11c.
Recall that we can rewrite
\[ 92_{\text{twelve}} \div E_{\text{twelve}} = ? \]
as
\[ 92_{\text{twelve}} = E_{\text{twelve}} \times ?. \]
Using the base-twelve multiplication table we look for \( 92_{\text{twelve}} \). We see that
\[ 92_{\text{twelve}} = E_{\text{twelve}} \times T_{\text{twelve}}, \]
so
\[ 92_{\text{twelve}} \div E_{\text{twelve}} = T_{\text{twelve}}. \]
as
Solution for Writing/Discussion (pg 179), 5.
The answer if found by adding
\[ 1 \times 51 + 2 \times 51 + 8 \times 51 + 16 \times 51. \]
By the distributive property, this equals
\[ (1 + 2 + 8 + 16) \times 51 = 27 \times 51 = 1377. \]
Notice that 27 = 11011\text{two}. In the “doubling” column, then umbers included in the sum are the 1s, and the numbers crossed out are the 0s.
Solution for Writing/Discussion (pg 179), 4.
You want to put the smaller number in the “halving” column if you want to do the computation in the least number of steps.

<table>
<thead>
<tr>
<th>Halving</th>
<th>Doubling</th>
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<tbody>
<tr>
<td>47</td>
<td>375</td>
</tr>
<tr>
<td>23</td>
<td>750</td>
</tr>
<tr>
<td>11</td>
<td>1500</td>
</tr>
<tr>
<td>5</td>
<td>3000</td>
</tr>
<tr>
<td>2</td>
<td>6000</td>
</tr>
<tr>
<td>1</td>
<td>12000</td>
</tr>
</tbody>
</table>

\[ 375 + 750 + 1500 + 3000 + 12000 = 17625 \]
The 2 and 6000 are crossed out. The sum of the remaining numbers in the “doubling” column is 17,625. Notice that 47 = 101111\text{two}, where each 1 represents a power of two multiplied by 375. The sum of these products is the answer.