1. An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. Such an example is called a *counterexample*. Provide a counterexample to show that each of the following statements if false.

(a) If \( x^2 = 49 \), then \( x = 7 \)

**Solution.** Let \( x = -7 \). Then \( x^2 = 49 \). This means that if \( x^2 = 49 \), then it isn’t necessarily true that \( x = 7 \).

(b) If a number is divisible by 4, then it is divisible by 6.

**Solution.** There are many counter examples. One would be the number 8, which is a number divisible by 4 but not divisible by 6.

(c) If \( ab < 0 \), then \( a < 0 \).

**Solution.** If we let \( b = -1 \) and \( a = 1 \), then \( ab < 0 \) but \( a > 0 \).

2. What can you conclude if the following statements are all true?

(i) If \( p \), then \( q \).  
(ii) \( p \)  
(iii) If \( q \), then not \( r \).  
(iv) \( s \) or \( r \).

**Solution.** Since \( p \) is true by (ii), then \( q \) must be true by (i). Now \( q \) being true implies that \( r \) is false by (iii). This implies that \( s \) is true by (iv), since either \( s \) or \( r \) must be true but \( r \) is not true.

All this means we can conclude that \( p, q, \) and \( s \) are true and \( r \) is false.

3. A tool for illustrating truth values of compound statements is a *truth table*. The following is an example of a truth table for the conditional statement “if \( p \), then \( q \)”.

<table>
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<th>( q )</th>
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(a) Given the conditional “if \( p \), then \( q \),” write out a truth table illustrating the truth values for the given conditional, its converse, its inverse, and its contrapositive.

**Solution.** This is a problem having to do with logical consistency, which can be tricky. Notice that in the truth table above that “if \( p \), then \( q \)” is false only when \( p \) is true and \( q \) is false. Thus this agrees with the meaning “if \( \ldots \), then \( \ldots \)” in promises. To see this you should try to think of an example. Such an example would be a person who promises, “If Lincoln was the second U.S. president, I’ll give you a dollar”. This person would not be a liar for failing to give you a dollar, since Lincoln was not the second president.

A curious consequence of the truth table for “if \( p \), then \( q \)” is that conditional statements may be true even when there is no connection between the hypothesis \( p \) and the conclusion \( q \). For example, the statement “If Kansas lies south of the
equator, then Paul Revere made plastic spoons” is true, since both statements are false.

You should work through an example to help you write out this truth table. The correct truth table can be found in your textbook in the chart at the bottom of page 860.

(b) Two statements are said to be logically equivalent when they have the same truth table. Which variant of the conditionals you found in part (a) are logically equivalent to the statement “p only if q”?

**Solution.** The statement “p only if q” is logically equivalent to “if p, then q”. For example, in the statement “A person is president of the United States only if the person well known” the hypothesis is “a person is president of the United States” and the conclusion is “the person is well known”. This is logically equivalent to “if a person is president of the United States, then that person is well known”.

4. The following argument is not logically valid, because it is missing a premise. Add premise that would make the argument complete and valid.

“I have $5.00 to spend for lunch. If the sandwich I want to buy costs $3.50, then I’ll have enough money left over to buy a beverage. If milk costs less than $1.50, I’ll buy it. The price of milk is $1.00. Therefore, I’ll buy milk with my sandwich.”

**Solution.** The sandwich costs $3.50.