1. Write a proof of the standard addition algorithm for the sum $74 + 67$.

**Proof.** Computing $74 + 67$ using the standard algorithm we get the following:

\[
\begin{array}{c}
1 \\
7 & 4 \\
+ & 6 & 7 \\
\hline
1 & 4 & 1 \\
\end{array}
\]

When summing by columns we are actually summing by place value, that is,

\[
74 + 67 = (70 + 4) + (60 + 7) \quad \text{Using the commutative and associative properties}
\]

\[
= (70 + 60) + (7 + 4) \quad \text{the first two steps say we can sum over the columns.}
\]

\[
= (70 + 60) + 11 \quad \text{In this step we sum the ones column.}
\]

\[
= (70 + 60) + 10 + 1 \quad \text{Here we regroup the 10 ones into 1 ten.}
\]

\[
= (10 + 70 + 60) + 1 \quad \text{This is the carrying step using the commutative and associative properties.}
\]

\[
= 140 + 1 \quad \text{The step is summing the tens column.}
\]

\[
= 141
\]

2. Create two three-digit numbers from the set \{4, 6, 7, 3, 5, 9\} to obtain the following. Use may use only use a number from the set once.

(a) The greatest difference between the two numbers.

**Solution.** To find the greatest difference you want the minuend to be the greatest number possible and the subtrahend to be the least number possible. To find the greatest number possible we take the three largest numbers from the set 6, 7, 9 and order the digits so that the greatest numbers are in the highest place values, that is, 976. To find the least number take the least three numbers 3, 4, 5 and order the digits so the smallest numbers are in the lowest place values, that is, 345. Doing this we get the result below.

\[
\begin{array}{c}
9 & 7 & 6 \\
- & 3 & 4 & 5 \\
\hline
6 & 3 & 1 \\
\end{array}
\]

(b) The least difference between the numbers.

**Solution.** To find the least difference we want the minuend and the subtrahend to be as close together as possible. Ordering the give set of numbers we get \{3, 4, 5, 6, 7, 9\}. First we look for the pair of numbers with the smallest values and smallest difference for our highest place value. The smallest difference between any pair is 1, so we choose the pair with the smallest numbers. In this case, we take the pair 3,4.

So far, our difference is

\[
\begin{array}{c}
4 & x & x \\
- & 3 & x & x \\
\end{array}
\]

With the remaining place values we choose the smallest two digit number to with the 4 and greatest to go with the 3. In our case we put 56 with the 4 and 97 with the 3. Then we get the result below

\[
\begin{array}{c}
3 & 14 \\
A & 5 & 6 \\
- & 3 & 9 & 7 \\
\hline
5 & 9 \\
\end{array}
\]
3. Diagnose what’s wrong with the following solution of a subtraction problem.

\[
\begin{array}{c}
4005 \\
- \quad 37 \\
\hline
2078
\end{array}
\]

If this was one of your student’s solution, how would you go about trying to help this student to subtract correctly?

**Solution.** It appears that the student tried to regroup the 4 twice, that is,

\[
\begin{array}{c}
\beta \\
A \quad 0 \quad 10 \quad 15 \\
\hline
2 \quad 0 \quad 7 \quad 8
\end{array}
\]

It is likely that this student unsure of what to do when there is a 0 in the next higher place value. You might reiterate what the student should do in this case.

\[
\begin{array}{c}
3 \quad 9 \quad 9 \\
A \quad 10 \quad 10 \quad 15 \\
\hline
3 \quad 9 \quad 6 \quad 8
\end{array}
\]