Part I. Multiple-Choice Questions (5 points each; please circle the correct answer.)

1. The graph in the $xy$-plane represented by $x = 3 \sin t$ and $y = 2 \cos t$ is
   (A) a circle
   (B) an ellipse
   (C) a hyperbola
   (D) a parabola
   (E) a line

2. The area enclosed by the polar equation $r = 4 + \cos \theta$, for $0 \leq \theta \leq 2\pi$, is
   (A) 0
   (B) $\frac{9\pi}{2}$
   (C) $18\pi$
   (D) $\frac{33\pi}{2}$
   (E) $\frac{33\pi}{4}$

3. Find the length of the arc of the curve defined by $x = \frac{1}{2}t^2$ and $y = \frac{1}{9}(6t + 9)^{\frac{3}{2}}$, from $t = 0$ to $t = 2$.
   (A) 8
   (B) 10
   (C) 12
   (D) 14
   (E) 16
4. The area of the region inside the polar curve \( r = 4 \sin \theta \) but outside the polar curve \( r = 2 \sqrt{2} \) is given by

\[
\text{(A) } 2 \int_{\pi/4}^{3\pi/4} (4 \sin^2 \theta - 1) \, d\theta \\
\text{(B) } \frac{1}{2} \int_{\pi/4}^{3\pi/4} (4 \sin \theta - 2 \sqrt{2})^2 \, d\theta \\
\text{(C) } \frac{1}{2} \int_{\pi/4}^{3\pi/4} (4 \sin \theta - 2 \sqrt{2}) \, d\theta \\
\text{(D) } \frac{1}{2} \int_{\pi/4}^{3\pi/4} (16 \sin^2 \theta - 8) \, d\theta \\
\text{(E) } \frac{1}{2} \int_{\pi/4}^{3\pi/4} (4 \sin^2 \theta - 1) \, d\theta .
\]

5. If, for \( t > 0 \), \( x = t^2 \) and \( y = \cos t^2 \), then \( \frac{dy}{dx} = \)

\[
\text{(A) } \cos t^2 \\
\text{(B) } - \sin t^2 \\
\text{(C) } - \sin 2t \\
\text{(D) } \sin t^2 \\
\text{(E) } \cos 2t
\]
Part II. Free-Response Questions

1. A particle moves in the $xy$-plane so that its position at any time $t$, for $-\pi \leq t \leq \pi$, is given by $x(t) = \sin 3t$ and $y(t) = 2t$.

   (a) (4 points) Sketch the path of the particle in the $xy$-plane provided. Indicate the direction of motion along the path.

   (b) (4 points) Find the range of $x(t)$ and the range of $y(t)$.

   (c) (4 points) Find the smallest positive value of $t$ for which the $x$-coordinate of the particle is a local maximum. What is the speed of the particle at this time?

   (d) (4 points) Write down an integral which will compute the distance traveled by the particle from $t = -\pi$ to $\pi$. 
2. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve $C$ is given by $x = \sqrt{1 + y^2}$. Let $S$ be the region bounded by the two graphs and the $x$-axis. The line and the curve intersect at point $P$.

(a) (5 points) Curve $C$ is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}$.

(b) (4 points) Use the polar equation given above to set up an integral expression with respect to the polar angle $\theta$ that represents the area of $S$. 
Name:________________________  Date: ______________  Period: _____
Part III. Multiple-Choice Questions (5 points each; please circle the correct answer.)

1. The length of the curve determined by $x = 2t^3$ and $y = t^3$ from $t = 0$ to $t = 1$ is

(A) $\frac{5}{7}$
(B) $\frac{\sqrt{5}}{2}$
(C) $\frac{3}{2}$
(D) $\sqrt{5}$
(E) 3

2. A particle moves along a path described by $x = \cos^3 t$ and $y = \sin^3 t$. The distance that the particle travels along the path from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 0.75
(B) 1.50
(C) 0
(D) $-3.50$
(E) $-0.75$

3. Find the area inside one loop of the curve $r = \sin 2\theta$.

(A) $\frac{\pi}{16}$
(B) $\frac{\pi}{8}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$
(E) $\pi$
4. A particle moves on a plane curve so that at any time \( t > 0 \) its position is defined by the parametric equations \( x(t) = 3t^2 - 7 \) and \( y(t) = \frac{4t^2 + 1}{3t} \). The acceleration vector of the particle at \( t = 2 \) is

(A) \( \left( 6, \frac{11}{12} \right) \)

(B) \( \left( 17, \frac{17}{6} \right) \)

(C) \( \left( 12, \frac{47}{12} \right) \)

(D) \( \left( 12, \frac{33}{12} \right) \)

(E) \( \left( 6, \frac{17}{6} \right) \)

5. The acceleration of a particle is described by the parametric equation \( x''(t) = \frac{t^2}{4} + t \) and \( y''(t) = \frac{1}{3t} \). If the velocity vector of the particle when \( t = 2 \) is \( (4, \ln 2) \), what is the velocity vector of the particle when \( t = 1? \)

(A) \( \left( \frac{5}{4}, \frac{1}{3} \right) \)

(B) \( \left( \frac{23}{12}, \frac{\ln 4}{3} \right) \)

(C) \( \left( \frac{23}{12}, \frac{\ln 2}{3} \right) \)

(D) \( \left( \frac{5}{4}, \frac{2}{3} \ln 2 \right) \)

(E) \( \left( \frac{23}{12}, \frac{4}{3} \ln 2 \right) \)
Part IV. Free-Response Questions

1. The figure to the right shows the path traveled by a roller coaster car over the time interval $0 \leq t \leq 18$ seconds. The position and velocity of the car at time $t$ can be modeled parametrically by

<table>
<thead>
<tr>
<th>Position</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(t) = 10t + 4 \sin t$</td>
<td>$x'(t) = 10 + 4 \cos t$</td>
</tr>
<tr>
<td>$y(t) = (20 - t)(1 - \cos t)$</td>
<td>$y'(t) = (20 - t) \sin t + \cos t - 1$</td>
</tr>
</tbody>
</table>

where $x$ and $y$ are measured in meters and $t$ is measured in seconds.

(a) **(4 points)** Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) **(4 points)** Find the time $t$ at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(c) **(4 points)** For $0 < t < 18$, there are two times at which at the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times.
2. The curve above is drawn in the \( xy \)-plane and is described by the equation in polar coordinates \( r = \theta + \sin(2\theta) \), \( 0 \leq \theta \leq \pi \), where \( r \) is measured in meters and \( \theta \) is measured in radians. The derivative of \( r \) with respect to \( \theta \) is given by \( \frac{dr}{d\theta} = 1 + 2 \cos(2\theta) \).

(a) (5 points) Find the area of the region below the curve, to the left of the \( y \)-axis, and above the \( x \)-axis.

(b) (4 points) For \( \frac{\pi}{3} < \theta < \frac{2\pi}{3} \), \( \frac{dr}{d\theta} \) is negative. What does this fact say about \( r \)? What does this fact say about the curve?

(c) (4 points) Find the value of \( \theta \) in the interval \( 0 \leq \theta \leq \frac{\pi}{2} \) that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.