Exercise Set 1 From Page 96 ff: 1-12, 15, 16, 19, 20 (do it this way: find a matrix \( x \) in the list and a matrix \( y \) in the list such that \( x^3 = y^2 = e \), \( yxy = x^2 \). Then simply verify that all six matrices are in the set \( \{ x^a y^b | 0 \leq a \leq 2, 0 \leq b \leq 1 \} \) ). 26-30, 35-38, 45.

Also try the following:

1. Let 
   \[ G = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{F}_2 \right\} . \]
   Show that \( G \) is a subgroup of \( GL_3(2) \). Show that \( G \) is generated by elements \( x, y \) satisfying \( x^4 = y^2 = e \), \( yxy = x^3 \).

2. Let \( G \) be a group and let \( \sigma, \tau \in G \) be two elements of order 2 such that \( o(\sigma \tau) = n \). Prove that \( |\langle \sigma, \tau \rangle| = 2n \). (Hint: if \( x = \sigma \tau \) and \( y = \sigma \), compute \( x^n, y^2 \) and \( yxy \). These familiar calculations should be helpful.)

3. We had a very short and informal discussion of generators and relations in class. In each of the following examples, prove that the groups presented are, in fact, trivial by showing that the generators themselves are trivial.
   (a) \( \langle x, y \mid xy = y^2 x, yx = x^2 y \rangle \).
   (b) \( \langle x, y \mid yx = x^2 y, xy^3 = y^2 x \rangle \).
   (c) \( \langle x, y \mid xy^2 = y^3 x, x^2 y = yx^3 \rangle \). (This is quite a bit tougher!)

4. If \( \sigma \in S_n \), let \( \text{Fix}(\sigma) = \{ i \mid 1 \leq i \leq n, \sigma(i) = i \} \). Now show that
   \[ \frac{1}{n!} \sum_{\sigma \in S_n} |\text{Fix}(\sigma)| = 1. \]
   In other words, each element of \( S_n \) fixes, on the average, only one element of the set \( \{1, 2, \ldots n\} \). (Hint: let \( \mathbb{N}_n = \{1, 2, \ldots n\} \) and consider the set
   \[ \{ (\sigma, i) \in S_n \times \mathbb{N}_n \mid \sigma(i) = i \} . \]
   Compute the order of this set in two ways.)
5. If you get the above result, try to prove the following generalization. Let $H \leq S_n$ be a transitive subgroup. Then

\[
\frac{1}{|H|} \sum_{\sigma \in H} |\text{Fix}(\sigma)| = 1.
\]

Exercise Set 2 From Page 110 ff: 1-13, 18, 19, 20, 25.