Puxi Campus High School Examinations  
Semester Two  
June 2010

IB Mathematics HL, Year 1  
(Paper 1)

Thursday, June 3rd, 2010  
12:45-2:00

Time: 1 hour, 15 minutes  

Mr. Surowski  

Student Name:__________________________

Instructions to the Candidate

• No food or drink to be brought into examination room.
• No talking during the examination.
• If you have a problem please raise your hand and wait quietly for a teacher.
• Please do not open the examination booklet until directed to do so.
• Please ensure that you have the correct examination in front of you.
• Write your name clearly in the space above when directed to do so.
• At the conclusion of your examination please refrain from speaking until you are outside the exam room as there may still be other examinations still in progress.
• Students are reminded that they are not permitted to leave the examination room early.

Special Instructions:

• Graphic calculators are not allowed on this paper.
• The exam has 12 pages including the cover page.

Good Luck!
Paper 1. 75 minutes; no calculators. Give exact answers where possible; otherwise, unless otherwise instructed, find solutions correct to three decimal places. (Total = 75 points)
1. Assume that $x$ and $y$ are related by the equation $y^2(5 - y) = x^4$. Find the possible values of $\frac{dy}{dx}$ where $y = 4$. 

Section A. Short-response questions. Each question is worth 6 marks.
2. Let $A$ and $B$ be events such that $P(A) = \frac{1}{5}$, $P(B \mid A) = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{10}$.

(a) Find $P(A \cap B)$.

(b) Find $P(B)$.

(c) Show that $A$ and $B$ are not independent events.
3. Compute the indefinite integral $\int \frac{\ln x \, dx}{x^3}$
4. \( R \) is the region in the first quadrant under the graph of \( y = \sin 3x, \ 0 \leq x \leq \frac{\pi}{4} \).

Find the \textbf{exact} volume of the solid formed by revolving \( R \) about the \( x \)-axis.
5. Box $A$ contains 6 red balls and 2 green balls. Box $B$ contains 4 red balls and 3 green balls. A cubical fair die with faces numbered 1, 2, 3, 4, 5, 6 is thrown. If 1 or 6 results, a ball is drawn from box $A$; otherwise a ball is selected from box $B$.

(a) Calculate the probability that a red ball will be selected.

(b) Given that a red ball is selected, calculate the probability that it comes from box $B$. 
6. Car A is traveling on a straight east-west road in a westerly direction at 60 km/hr. Car B is traveling on a straight north-south road in a northerly direction at 70 km/hr. The two roads intersect at the point \(O\), as indicated in the diagram to the right.

Find the rate of change of the indicated variable \(z\) when Car A is 0.8 km east of \(O\) and Car B is 0.6 km south of \(O\).
7. Let \( f(x) = \frac{x^2 + 5x + 5}{x + 2}, \quad x \neq -2. \)

(a) Find \( f'(x). \)

(b) Solve the inequality \( f'(x) > 0. \)
1. Assume that there is a large box containing \( m \) white ping-pong balls and \( 2m \) orange ping-pong balls. We shall be taking ping-pong balls from this box; let \( X \) be the number of white balls selected.

(a) (3 points) Assume that you select \( k \) ping-pong balls from the box, with replacement. Compute \( P(X \geq 2) \) as a function of \( k \).

(b) (1 point) Now assume that you select exactly three balls, again with replacement, and compute \( P(X = 2) \).

(c) (3 points) Select three balls from the above box, without replacement, and show that \( P(X = 2) = \frac{2m(m-1)}{(3m-1)(3m-2)} \).

(d) (2 points) Let \( f(m) = \frac{2m(m-1)}{(3m-1)(3m-2)} \) and show that \( \lim_{m \to \infty} f(m) = \)

(e) (2 points) Why should this be?
2. You are given the graph of \( y = 1 + \frac{1}{x}, \quad x > 0 \). The region \( R \) is bounded by the above graph, the \( y \)-axis, and the lines \( y = 2 \) and \( y = b \), where \( b > 2 \) is a constant.

(a) (5 points) Let \( A \) denote the area of the region \( R \) and solve the equation \( A = \frac{1}{2} \) for \( b \).

(b) (5 points) The region \( R \) is revolved about the \( y \)-axis; let \( V \) denote the volume of the solid so obtained. Solve \( V = \frac{\pi}{2} \) for \( b \).
3. You are given the diagram to the right.

**(5 points)** Show that

\[
\theta = \arctan \frac{2}{x} - \arctan \frac{1}{x}.
\]

(b) **(7 points)** Hence, or otherwise, find the exact value of \(x\) for which \(\theta\) is a maximum.
High School Examinations
Semester Two 2008

IB Mathematics HL, Year 1
(Paper 2)

Friday, June 6, 2008
2:15—3:45 P.M

Time: 1.5 hours

Teacher: Mr. Surowski

Student Name:___________________________

Instructions to the Candidate
• No food or drink to be brought into examination room.
• Please do not talk during the examination.
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• Please ensure that you have the correct examination in front of you.
• Write your name clearly in the space above when directed to do so.
• At the conclusion of your examination please refrain from speaking until you are outside the exam room as there may still be other examinations still in progress.
• Students are reminded that they are not permitted to leave the examination room early.

Special Instructions:
• Examination must be completed in pen.
• Graphic Calculators are allowed on this paper.
• Please write answers in spaces provided.

Good Luck!
Name: ___________________________ Date: ________ Period: ___

Paper 2. 75 minutes; calculators allowed. Give exact answers where possible; otherwise, unless otherwise instructed, find solutions correct to three decimal places. Solutions found from a graphic display calculator should be supported by suitable working, e.g., if graphs are used to find a solution, you should sketch these as part of your answer. (Total = 78 points)
1. You are given the graph of

\[ y = (x^3 - 3x - 1) \sin x, \]
as indicated to the right.

(a) The graph of \( y = f(x) \) has \( x \)-intercepts at the indicated points \( A \) and \( B \). Determine the \( x \)-coordinates of these two points.

(b) Compute the area of the shaded region.
2. (a) Using integration by parts, compute $\int x \cos x \, dx$.

(b) The region $R$ bounded by the curve $y = \cos x$, $0 \leq x \leq \frac{\pi}{2}$, and the $x$- and $y$-axes. Using your result of (a) or otherwise, find the exact volume of the solid obtained by revolving $R$ about the $y$-axis.
3. A box contains 4 red balls and \( n \) green balls. Randomly choose three balls from this box, without replacement, and let \( X \) be the number of red balls selected. Assuming that \( P(X = 2) = \frac{14}{55} \), compute \( n \).
4. Shaft $AB$ is 30 cm long and is attached to a flywheel at $A$. $B$ is constrained to motion along $OX$. The radius of the wheel is 15 cm, and the wheel rotates at 100 revolutions per second.

Find the rate of change in angle $\angle ABO$ when angle $\angle AOX$ is $120^\circ$. 
5. A particle moves along a straight line with velocity given by \( v(t) = \frac{1}{2 + t^2} \), measured in meters per second.

(a) Find the total distance traveled in the first two seconds.

(b) Find an expression for the acceleration at time \( t \).
6. Compute the maximum value of the function \( f(x) = x^2 e^{-x/10} \) on the interval \( 0 \leq x < \infty \), and explain why your computed value is the maximum.
7. Use integration by parts to compute \( \int_0^m \sin^{-1} x \, dx \), giving your answer as a function of \( m \).
1. The function \( g \) is defined by setting \( g(x) = \frac{e^x}{\sqrt{x}}, \quad 0 < x \leq 3. \)

(a) (2 points) Sketch the graph of \( y = g(x) \).

(b) (2 points) Compute \( g'(x) \).

(c) (2 points) Write down an expression representing the gradient of the normal to the curve at any point.

Assume now that \( P \) is a point \((x, y)\) on the graph of \( g \), and that \( Q = Q(1, 0) \).

(d) (3 points) Find the gradient of the line \((PQ)\) in terms of \( x \).

(e) (3 points) Find the value of \( x \) for which the line \((PQ)\) is normal to the graph of \( g \). Use this result to determine the distance from the graph of \( g \) to the point \( Q \).
2. Two women, Ann and Bridget, play a game in which they take turns throwing an unbiased six-sided die. The rules are as follows:

- anyone who throws a “1” automatically loses;
- anyone who throws a “6” automatically wins;
- if someone throws anything other than a 1 or a 6, the player gives the die to the other player, and the game continues.

Ann is the first to throw the die.

(a) Let $X$ be the number of rounds in the game (the tosses of the die). Therefore, $X = 2$ means that the game ends on the second toss of the die. Find

(i) (2 points) $P(X = 2)$

(ii) (2 points) $P(X \leq 3)$.

(iii) (3 points) Compute the conditional probability $P(\text{Ann wins} | \text{game lasts no more than three rounds})$.

(b) (5 points) Compute the probability that Ann wins the game.
3. The diagram to the right depicts the graphs of \( y = \ln x \) and \( y = mx \), where \( m \) is a constant.

(a) (4 points) Find the value of \( m \) such that \( y = mx \) is tangent to the graph of \( y = \ln x \).

(b) (3 points) For the value of \( m \) found in part (a), find the coordinates of the point of tangency.

(c) (5 points) Compute the area of the region \( R \) bounded by the \( x \)-axis and the two graphs.