Mathematics for Elementary School Teachers

Exam 1
February 24, 2003

The point value of each problem is given in the margin.

(8) 1. Identify each of the following sequences as arithmetic, geometric or neither and answer the questions asked.

(a) A student collecting bottle caps started with 7 on the first day and added 5 more each day thereafter. Write the first 5 terms of the sequence: 7, 12, 17, 22, 27. What type of sequence is it? **Arithmetic** How many bottle caps will the student have on the 100th day?

\[ 7 + 99 \cdot 5 = 7 + 495 = 502 \]

(b) 5, 15, 45, 135, .... What type of sequence is it? **Geometric** Find the 17th term of the sequence, expressing your final answer using exponent notation.

\[ 5, 5 \cdot 3, 5 \cdot 3^2, 5 \cdot 3^3, ... \]

17th term = \( 5 \cdot 3^{16} \)

(6) 2. Let the universe \( U = \{1, 2, 3, 4, ..., 21\} \), \( F = \{1, 2, 3, 5, 8, \ldots\} \), the set of Fibonacci numbers (without repetition of 1), and \( T = \{1, 3, 6, 10, 15, 21\} \), the set of triangular numbers. Draw a Venn diagram illustrating the two sets \( F, T \) inside the universe \( U \). (Every element of \( U \) should be placed in your diagram.)

(8) 3. Let \( A = \{a, b, c, d, e, f, g\} \), \( B = \{d, e, f, g, h, i, j, k, l, m\} \), \( C = \{h, e, h, j\} \).

(a) Find the set difference \( C \setminus A \)

\[ \{h, e, h, j\} \setminus \{a, b, c, d, e, f, g, h, i, j, k, l, m\} = \{h, e, h, j\} \]

Delete from \( C \) any element of \( A \).

(b) Find \( A \cup (B \cap C) \)

\[ A \cup (B \cap C) = \{a, b, c, d, e, f, g\} \cup \{h, e, h, j\} = \{a, b, c, d, e, f, g, h, i, j\} \]

(8) 4. In a class of 36 students, 16 watched television last night, 22 did homework, 12 had pizza, 11 watched television and did homework, 7 watched television and ate pizza, 9 ate pizza and did homework and 5 did all three. Illustrate this information on a Venn diagram using \( T \) for students who watched television, \( H \) for those who did homework and \( P \) for those who ate pizza. (Note: When we say 11 students watched television and did homework, some of those students also ate pizza. Likewise for the other numbers.)

How many students did none of these activities?

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(8) 5. a) Write the following as a base-10 numeral.
\[ 7 \cdot 10^6 + 3 \cdot 10^5 + 3 = 7,000,303 \]
b) Write the following base-10 numeral in expanded form.
\[ 934907 = 3 \cdot 10^6 + 3 \cdot 10^4 + 4 \cdot 10^3 + 7 \]

(6) 6. Recall the Roman numerals \( L = 50 \), \( C = 100 \), \( D = 500 \), \( M = 1000 \).
(a) Convert 698 into Roman numerals.
\[ \text{DCXCV} = \frac{500}{600} + \frac{100}{90} + 8 \]
(b) Identify the year MCDXCII.
\[ 1000 + 900 + 90 + 2 = 1492 \]

(6) 7. Express the following binary number in base-10.
\[ 1100101_{\text{two}} = 64 + 32 + 4 + 1 = 101 \]
\[ 101 \]

(6) 8. Express the base-10 numeral 200 in base-5.
Place values: 1, 5, 25, 125
\[ 200 = 1 \cdot 125 + 3 \cdot 25 + 0 \cdot 5 + 0 \cdot 1 = 1300_{\text{five}} \]

(6) 9. Fill in the blank and indicate the law illustrated. (Spelling counts!)
(a) \( 6(bc) = (6b)c \)
\[ \text{associative law (for multiplication)} \]
(b) \( 3a - 25a = (3 + 25)a \)
\[ \text{distributive law} \]
(c) \( 42 + 13 + 17 = 42 + 17 - 13 \)
\[ \text{commutative law (for addition)} \]

(9) 10. Determine whether the following sets are closed under the given operation. If not give a counterexample.
(a) \{ -1, 0, 1 \} under addition. Not closed, \( 1 + 1 = 2 \)
(b) \{0, 1, 2, 3, 4, 5, 7, 8, ... \}, the set of whole numbers without 6, under multiplication.
\[ \text{Not closed. } 2 \cdot 3 = 6 \]
(c) \{7, 9, 11, 13, ... \}, the set of odd whole numbers \( \geq 7 \), under multiplication.
\[ \text{Yes closed. } \text{The product of two odds is odd.} \]
\[ \text{The product of two numbers } \geq 7 \text{ is } \geq 7. \]
(8) 11. A function \( f(x) \) is given by the set of ordered pairs
\[ \{(2, 5), (3, 7), (4, 9), (6, 13)\} \].
(a) What is the domain of the function \( f(x) \)?
\[ 2, 3, 4, 6 \]
(b) Give a formula for the function \( f(x) \). (Hint: It's of the form \( f(x) = ax + b \);
\[ f(x) = 2x + 1 \]
(c) Draw an arrow diagram for \( f(x) \).

(5) 12. A student is trying to understand why we add exponents when multiplying two quantities with the same base: \( b^m b^n = b^{m+n} \). Explain it using a good example.
\[ 2^3 \cdot 2^5 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \]
\[ = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{3 + 5 + \text{twos}} = 2^{3+5} = 2^8 \]

(8) 13. Calculate.
a) \( 8 - 2 + 3 = 9 \)  
b) \( 5 - 2(6 - 1) = 5 + 2 \cdot 5 = 15 \)
c) \( (1 + 2)^2 = 9 \)  
d) \( 3 \cdot 2^3 = 3 \cdot 8 = 24 \)

(4) 14. Give the quotient and remainder or state that they do not exist. Also show how to write your answers in the form \( a = qb + r \):
\[ 42 \div 5 \]
\[ \frac{8}{42} \]
\[ q = 8 \]
\[ r = 2 \]

(4) 15. a) What are the possible remainders in dividing two whole numbers if the divisor is 6?
\( 0, 1, 2, 3, 4, 5 \)
b) What are the possible remainders if the dividend is 6? (Hint: Test \( 6 \div 1, 6 \div 2, \ldots \))
\[ \frac{1}{6} \]
\[ \frac{2}{6} \]
\[ \frac{3}{6} \]
\[ \frac{4}{6} \]
\[ \frac{5}{6} \]
\[ \frac{6}{6} \]
\[ \frac{7}{6} \]
\[ \frac{8}{6} \]
\[ \frac{9}{6} \]
\[ \frac{0}{6} \]
\[ \text{Always 6 from here on} \]