Mathematics for Elementary School Teachers
Final Exam
December 17, 2004

The point value of each problem is given in the margin. You must show all your work to receive full credit.

(8) 1. Identify each of the following sequences as arithmetic, geometric or neither and answer the questions asked.
   (a) Your salary this year (2004) is $40000 and it increases by $500 each year thereafter. Type of sequence? arithmetic What is your salary in the year 2023?

   \[
   \frac{500}{200} \quad \frac{40000}{95000}
   \]

   \[
   40000 + 19 \cdot 500 = \$49500
   \]

   (b) The current value of your home is $60000 and it is increasing in value by 7\% each year, that is, the value increases by a factor of 1.07 each year. Type of sequence? geometric

   What is the value 20 years from now? (expressed in exponent notation.)

   \[
   60000 \cdot (1.07)^{20}
   \]

(8) 2. Draw a Venn diagram to illustrate the three sets A, B, C below inside the universal set

   \[
   \begin{align*}
   U &= \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\} . \\
   A &= \text{the factors of 30 in } U \\
   B &= \text{all multiples of 5 in } U \\
   C &= \text{all primes in } U
   \end{align*}
   \]

(8) 3. An automobile shop has 50 cars with problems. There are 18 cars with flat tires, 14 needing oil, 10 with broken mufflers, 3 with broken mufflers needing oil, 8 with flat tires needing oil, 5 with flat tires and broken mufflers, and 2 cars with flat tires, broken mufflers, needing oil. (Note, the word "only" has not been used. Thus, when we say there are 3 cars with broken mufflers needing oil, some of those cars may have flat tires as well.)

   (a) Illustrate this information on a Venn Diagram. Use the labels:

   \[
   M = \text{Broken Mufflers, } L = \text{Need Oil, } F = \text{Flat Tires.}
   \]

   (b) How many cars in the shop don’t have any one of the three problems listed above?

   22
(9) 4. a) Write the following as a decimal. \(10^5 + 2 \cdot 10^2 + 10^{-1} + 3 \cdot 10^{-2} = 100200.13\)

b) Express the following number in words: 500000.0304

Five hundred thousand and three hundred thousandths

c) Arrange the numbers 4/3, 1.33, \(\sqrt{2}\) from smallest to largest. (Hint: If \(x^2 < y^2\) then \(x < y\).)

\(\frac{1}{3} = 0.333\ldots, \quad \text{Note} \quad \left(\frac{4}{3}\right)^2 = \frac{16}{9}, \quad \left(\sqrt{2}\right)^2 = 2, \quad \text{Thus} \quad 1.33 < \frac{4}{3} < \sqrt{2}\)

(5) 5. Express the following binary number in base-10: \(1001100_{\text{two}} = 64 + 8 + 4 = 76\)

\(111_{\text{two}} = 6 + 4 = 10\)

(6) 6. Express the base-10 numeral 333 in base-5. Place values 1, 5, 25, 125

\[
333 = 2 \cdot 125 + 3 \cdot 25 + 1 \cdot 5 + 3 \cdot 1 = 2313\]

\[
\begin{array}{c|cccc}
3 & 3 & 3 & \div & 5 \\
2 & 5 & 0 & \div & 5 \\
2 & 5 & \div & 5 & 0
\end{array}
\]

(9) 7. Determine whether the following sets are closed under the given operation. If not give a counterexample.

(a) The set of primes under addition. Not closed \(3 + 5 = 8\)

(b) \(\{0, 1, 2, 3, 4, 5, 6, 8, 9, \ldots\}\), the set of whole numbers without 7 under multiplication.

Closed

(c) The set of negative integers under subtraction. Not closed

\((-1) - (-2) = -1 + 2 = 1\)

(10) 8. Estimate. (Do not calculate exact values.)

(a) \(\frac{360011}{9.02} \approx \frac{360000}{9} = 40000\)

d) \((3 + \frac{1}{8})^3 + (1.99)^2 \approx 3 + \frac{2}{2} = 27 + 4 = 31\)

(b) \(6700013 - 225 \approx 6700000\)

e) 9.8% of 14.98 \(\approx 10\% \text{ of } 15\)

\(= 1.5\)

(c) \(\frac{901}{899} + \frac{201}{51} + \frac{512}{97} \approx 1 + 4 + \frac{1}{2} = 5\frac{1}{2}\)

(6) 9. A student is trying to understand why we add exponents in the law of exponents \(b^m \cdot b^n = b^{m+n}\). Explain it using a good example.

\(b^2 \cdot b^3 = (b \cdot b) \cdot (b \cdot b \cdot b) = b^2 \cdot b^3 = b^{2+3} = b^5\)
(9) 10. Use tests to determine whether 20030082 is divisible by the following numbers. (Show clearly how the test works.)

a) 4 \[ \text{Test 4: } 4 \mid 82 \text{ so } 4 \mid 20030082 \]

b) 6 \[ \text{Test 2, 3: } \text{Even \# so 2 is a divisor.} \\
2 + 3 + 0 + 8 + 2 = 15, \quad 3 \mid 15, \quad \text{so } 3 \mid 20030082. \text{ Thus } 6 \mid 20030082 \]

c) 11 \[ 2 \cdot 0 + 0 - 0 + 0 - 8 + 2 = 2 + 3 + 2 - 8 = -1, \quad 11 \nmid -1 \text{ so } 11 \nmid 20030082 \]

(6) 11. Calculate 5.04 ÷ .03 using the standard long division algorithm and explain how and why the algorithm works.

\[ \begin{array}{c|cccc}
\text{0.03} & 5.04 \\
\hline
\text{3} & 5.04 \\
\hline
\text{204} & \text{504} \\
\hline
\text{18} & \text{50} \\
\hline
\text{24} & \text{0} \\
\hline
\end{array} \]

\[ 5.04 = \frac{504}{100} = \frac{504}{3}, \text{ so we move decimal point over two places and calculate } 504 ÷ 3 \]

to get 168.

(6) 12. Find the greatest common factor of 2323 and 2300 any way you like.

\[ \text{GCF (2323, 2300)} \]
\[ = \text{GCF (23, 2300)} \]
\[ = \text{GCF (23, 0)} \]
\[ = 23 \]

(5) 13. Find the least common multiple of 45 and 100 any way you like.

\[ \text{LCM (45, 100)} = 2^2 \cdot 3^2 \cdot 5^2 = 4 \cdot 9 \cdot 25 = 900 \]

(5) 14. Convert 2.1\overline{15} to an improper fraction in simplified form.

\[ x = 2.1515\ldots \]
\[ 100x = 215.1515\ldots \]
\[ 99x = 213 \]

\[ x = \frac{213}{99} = \frac{71}{33} \]

(5) 15. A recipe calls for 3 cups of flour and 2 cups of milk. If you only have 2 cups of flour how much milk should you add? (Set up a proportion and solve it.)

\[ \frac{\text{Milk}}{\text{Flour}} = \frac{2}{3} = \frac{x}{2} \quad \Rightarrow \quad 4 = 3x \quad \Rightarrow \quad x = \frac{4}{3} \text{ cups milk} \]
(4) 16. The radius of a circle is doubled.
   (a) By what factor is the circumference increased by? \( \frac{2}{1} \) \( C = 2\pi r \), \text{after doubling } r, \quad C = 4\pi r 
   (b) By what factor is the area increased by? \( \frac{4}{1} \) \( A = \pi r^2 \), \text{after doubling } r, \quad A = \pi (2r)^2 = 4\pi r^2 

(6) 17. a) One cubic yard of peatmoss is how many cubic feet?
   \( 1 \text{ yd}^3 = 3 \text{ ft}^3 \), \( 1 \text{ yd}^3 = 3^3 \text{ ft}^3 = 27 \text{ ft}^3 \)
   b) 1.25 meters is how many centimeters?
   \( 1.25 \text{ m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 125 \text{ cm} \)

(6) 18. 15% of the students in a class were men.
   a) What fraction of the class were women?
   \( 85% = \frac{85}{100} = \frac{17}{20} \)
   b) What was the ratio of men to women in the class? \( 3:17 \) or \( 3/17 \)

(6) 19. Draw the reflection axes and indicate the rotation symmetry for the following shapes.
   a) [Diagram of a rectangle with 180° or 2-turn rotation symmetry]
   b) [Diagram of an equilateral triangle with 120° or 3-turn rotation symmetry]

(6) 20. A rectangular box has edges of lengths 2, 3 and 4 inches.
   (a) What is the volume?
   \[ V = 2 \cdot 3 \cdot 4 = 24 \text{ in}^3 \]
   (b) What is the surface area?
   \[ 2 \left( 2 \cdot 3 + 2 \cdot 4 + 3 \cdot 4 \right) = 2 \left( 6 + 8 + 12 \right) = 2 \cdot 26 = 52 \text{ in}^2 \]

(6) 21. A home valued at $50,000 went up in value 10% the first year and down 10% the second year. What was its value at the end of the second year?
   After one year: \( 50,000 + 5,000 = 55,000 \)
   After two years: \( 55,000 - 5,500 = 49,500 \)
   \( \frac{55,000}{49,500} \)
22. Simplify. a) \(1^{-1} + 2^{-1} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = \frac{3}{2}\)

b) \(7 - 2(1 - 3) = 7 - 2(2) = 7 + 4 = 11\)

c) \((-1 - (-1)^2)^3 = (-1 - 1)^3 = (-2)^3 = -8\)

23. Let \(N=\text{natural numbers, I=integers, Q=rationals and R=reals}\). Indicate all of the sets in the previous sentence that each of the following numbers belong to.

a) \(\sqrt{18} = 3\sqrt{2}\) \(\text{in } R\)

b) \(3\frac{1}{7}\) \(\text{I, Q, R}\)

c) \(3.0\overline{5} = 3.0505050...\) \(\text{Q, R}\)

d) \(-7\) \(\text{I, Q, R}\)

24. (a) State the associative law for addition. \((a+b)+c = a+(b+c)\) for any real numbers \(a, b, c\)

(b) A student writes \(8 - 3 + 4 = 1\). What error was made? The student added the 3 and 4 first and then subtracted it from 8. It should be \(8 - 3 + 4 = 9\).

Why do you think the student made the error? May have been thinking about PEMDAS, and added before subtracting.

25. Define and draw a picture to illustrate.

a) Rhombus. Parallellogram with all sides of the same length.

b) Acute triangle. Triangle with 3 acute angles.

26. Find the area of the triangle pictured below.

\[5^2 + h^2 = 6^2\]

\[\Rightarrow h^2 = 36 - 25 = 11\]

\[h = \sqrt{11}\]

Area = \(\frac{1}{2}\) base \cdot height

\[= \frac{1}{2} \cdot 10 \cdot \sqrt{11}\]

\[= 5\sqrt{11}\]
(9) 27. Find the missing angles A, B in the following figures.

\[ A = 60^\circ \]

\[ B = 45^\circ \]

(5) 28. Given \( F = \frac{9}{5}C + 32 \) and that the current temperature is \( 50^\circ F \), find the temperature in degrees C.

\[ 50 = \frac{9}{5}C + 32 \quad \Rightarrow \quad \frac{9}{5}C = 18 \quad \Rightarrow \quad C = \frac{5}{9} \cdot 18 = 10^\circ \]

(6) 29. Give estimates for the following.

(a) 8 liters \( \approx 2 \) gallon(s)

(b) 10 meters \( \approx 30 \) feet

(c) 50 kilograms \( \approx 100 \) pounds

(5) 30. The speed of light is about 300 million meters per second. Estimate how many miles per hour this is given that 1 mile is about 1600 meters and 1 hour is 3600 seconds.

\[
300 \times 10^6 \text{ m/sec} \times \frac{1 \text{ mi}}{1600 \text{ m}} \times \frac{3600 \text{ sec}}{1 \text{ hr}}
\]

\[
= \frac{75}{360} \cdot \frac{3.0 \times 10^6}{16} \text{ mi/hr}
\]

\[
= \frac{75}{675} \cdot 10^6 \text{ mi/hr}
\]

\[
= 675 \times 10^6 \text{ mi/hr}
\]

\[
= 675 \text{ million mi/hr}
\]