Mathematics for Elementary School Teachers
Final Exam
May 9, 2001

The point value of each problem is given in the margin.

(8) 1. Identify each of the following sequences as arithmetic, geometric or neither and answer the questions asked.

(a) Postage in a certain country was 10 cents the first year and increased by two cents a year for the next 80 years. Type of sequence? arithmetic. What was the postage in the 71-st year? \(10, 12, 14, 16, \ldots\)
\[ P_n = 10 + (n-1)2, \quad P_{71} = 10 + (71-1)2 = 10 + 70 = 80 \]

(b) A gallon of ice-cream is consumed in such a manner that at the end of the first day \(\frac{1}{2}\) gallon is left and the amount left at the end of each subsequent day is half of the amount left from the previous day. Type of sequence? geometric.
How much ice-cream is left after 5 days (in gallons)? \(\frac{1}{32}\) gallon.
\[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \]

(5) 2. Make a Venn diagram to illustrate \(A \cap B\)

(6) 3. Let \(A = \{x | x > 0 \text{ and } x \text{ is odd}\}\), \(B = \{x | x > 6\}\), \(C = \{2, 4, 5, 7, 8\}\).

(a) Find \(C - A\) \(= \{3, 5, 7, 8\}\)

(b) Find \((A \cup B) \cap C\) \(= \{3, 5, 7, 8\}\)

(8) 4. a) Write the following as a decimal. \(4 \times 10^5 + 2 \times 10^3 + 3 \times 10^{-2}\) = 402000.03

b) Write the following decimal in expanded form. 20030.005
\[ = 2 \times 10^4 + 3 \times 10 + 5 \times 10^{-3} \]

(5) 5. Express the following binary number in base-10.

\[ 1000101_{two} = 64 + 4 + 1 = 69 \]
6. Express the base-10 numeral 200 in base-5. Place values 1, 5, 25, 125

\[ 200 = 4 \cdot 25 + 3 \cdot 5 + 0 \cdot 1 = 1300_{\text{five}} \]

7. Determine whether the following sets are closed under the given operation. If not give a counterexample.

(a) The set of rational numbers under subtraction.
Yes

(b) \{0, 1, 2, 3, 4, 5, 6, 8, 9, \ldots \}, the set of whole numbers without 7 under addition.
No, 3 + 4 = 7

(c) The set of negative integers under multiplication?
No, \((-2) \cdot (-3) = 6\)

8. Estimate. (Do not calculate exact values.)

(a) \(\frac{45011}{91} \approx 490 \approx 500\)

(b) \(21000813 - 759 \approx 21000000\)

(c) \(\frac{1}{99} + \frac{99}{97} + \frac{60}{29} \approx 0 + 1 + 2 = 3\)

9. A student is trying to understand why we subtract exponents in the law of exponents \(\frac{b^n}{b^m} = b^{n-m}\). Explain it using a good example.

\[ \frac{b^6}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^{6-2} = b^4 \]

10. Use tests to determine whether 3122112 is divisible by the following numbers. (Show clearly how the test works.)

(a) 4
Yes, because \(4 \mid 12\)

(b) 3
\(3 + 1 + 2 + 2 + 1 + 1 + 2 = 12\), \(3 \mid 12\) so Yes

(c) 9
\(9 \mid 12\) so No
(6) 11. a) Find the prime factorization of 400.

\[ 400 = 4 \times 100 = 2^2 \times 10^2 = 2^2 \times 2^2 \times 5^2 = 2^4 \times 5^2 \]

b) Determine the number of positive divisors of 400.

\[ (4+1)(2+1) = 15 \]

(6) 12. Given that \( 5625 = 3^2 \cdot 5^4 \), \( 4725 = 3^3 \cdot 5^2 \cdot 7 \) find

\[(i) \quad \text{GCF}(5625, 4725) = 3^2 \cdot 5^2 = 9 \cdot 25 = 225 \]

\[(ii) \quad \text{LCM}(5625, 4725) = 3^3 \cdot 5^4 \cdot 7 \quad \text{Expressed in factored form} \]

(5) 13. State the Fundamental Theorem of Arithmetic.

Any composite number can be expressed in one and only one way as a product of primes, (aside from the order of the primes.).

(5) 14. Order the decimals from smallest to largest.

\[
0.51, \quad 0.5151, \quad 0.515, \quad 0.515, \quad \frac{5151}{515}, \quad 0.5151, \quad 0.5151, \quad 0.5151 \]

\[
.5151 < .51 = .515 < .515 \]

\[
.515 = .515151515 \]

(8) 15. Calculate and express your final answer in simplified form.

\[(a) \quad \frac{7}{15} \div \frac{28}{5} = \frac{7}{3} \times \frac{5}{28} = \frac{1}{12} \]

\[(b) \quad \frac{7}{45} - \frac{1}{6} = \frac{14}{90} - \frac{15}{90} = -\frac{1}{90} \]

(5) 16. Calculate and express your final answer in scientific notation.

\[
\frac{2.4 \times 10^7}{6 \times 10^{-3}} = \frac{24 \times 10^6}{6 \times 10^{-3}} = 4 \times 10^{6-(-3)} = 4 \times 10^9 \]
(8) 17. Calculate exactly
   a) \(0.14 \times 15.5\)
   
   \[
   \begin{array}{c}
   0.14 \\
   \times 15.5 \\
   \hline
   2170 \\
   \hline
   \end{array}
   \]
   
   \[
   92\%
   \]

   b) \(3.12 \div 0.18\)
   
   \[
   \begin{array}{c}
   3.12 \\
   \div 0.18 \\
   \hline
   0.18 \\
   \hline
   132 \\
   \hline
   126 \\
   \hline
   60 \\
   \hline
   60 \\
   \hline
   60 \\
   \hline
   60 \\
   \hline
   \end{array}
   \]
   
   \[
   66\%
   \]

(5) 18. Four-fifths of all the students in a class had finished their exams.
   a) What is the ratio of those who had finished to those who had not? \(4:1\)
   
   \[
   77\%
   \]

   b) What percentage of students had not yet finished? \(\frac{4}{5} = 0.8 = 80\%\)
   
   \[
   88\%
   \]

(5) 19. Convert the decimal 3.12 to a mixed fraction in simplified form.
   
   \[
   81\%
   \]

   \[
   3.12 = 3 + \frac{12}{100} = 3 + \frac{6}{50} = 3 + \frac{3}{25} = 3\frac{3}{25}
   \]

(5) 20. A recipe calls for 2 cups of flour and \(1/3\) cup of milk. If you only have \(1\frac{2}{3}\) cups of flour, how much milk should you add?
   
   \[
   \frac{Milk}{Flour} = \frac{\frac{1}{3}}{2} = \frac{x}{1\frac{2}{3}} \Rightarrow \frac{1}{3} \times \frac{5}{3} = 2x \Rightarrow \frac{5}{9} = 2x \Rightarrow x = \frac{5}{18} \text{ cups}
   \]

(5) 21. The morning temperature was \(50^\circ\). It went up 40 percent by mid-afternoon and then down 20 percent (from the mid-afternoon temp.) by early evening. What was the early evening temperature?
   
   \[
   50 + (\frac{4}{10}) \times 50 = 50 + 20 = 70^\circ \text{ mid-afternoon}
   \]

   \[
   70^\circ - (\frac{2}{10}) \times 70 = 70 - 14 = 56^\circ \text{ evening}
   \]
22. Simplify. 
   a) \((1/4)^{-1} = 4\)  
   b) \((-1)^{-3} = \frac{1}{(-1)^3} = \frac{1}{-1} = -1\) 
   c) \((1 + (-1)^4)^{2} = (1 + 1)^2 = 2^2 = 4\)

23. a) Draw a Venn diagram to illustrate the relationship between the sets of 
    \(N = \text{natural numbers}, I = \text{integers}, Q = \text{rationals and} R = \text{reals}\). 

   ![Venn Diagram](image)

   b) Give an example of a number in each of the following sets: 
   \(\sqrt{2} \in Q - I, 5 \in Q \cap I \cap N, \pi \in R - Q\)

24. How can you tell by looking at a fraction \(a/b\) (in simplified form) whether 
   its decimal expansion is repeating or terminating?
   
   It terminates only if \(b = \pm 1\) or the only prime factors of \(b\) are \(2\) and/or \(5\).

25. Define and draw a picture to illustrate. 
   a) Scalene Triangle: \(\text{Triangle with all three sides of different lengths}\)

   ![Scalene Triangle]

   b) Trapezoid: \(\text{Quadrilateral having exactly one pair of opposite sides that are parallel}\)

   ![Trapezoid]

   c) Kite: \(\text{Quadrilateral with two pairs of adjacent sides of the same lengths}\)

   ![Kite]

26. Find the missing angles \(A, B, C\) in the following figures.
   
   ![Triangle with angles](image)
   \(2A = 160\)  
   \(A = 80\) \(90\%\)  
   \(3C = 180\)  
   \(C = 60\) \(85\%\)  
   \(B = 70\) \(80\%\)
(6) 27. Draw a right triangle with legs of lengths 3 and 6 respectively and then find the length of the hypotenuse. Express your answer in simplified form.
\[ x^2 = 3^2 + 6^2 = 9 + 36 = 45 \]
\[ x = \sqrt{45} = 3\sqrt{5} \]

(9) 28. Convert the following. a) 8 pints to gallons. \[ 8 \text{ pts} = 4 \text{ pts} = 1 \text{ gal}. \]

b) \(6 \frac{2}{3}\) yards to feet. \[ \frac{20}{3} \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} = 20 \text{ ft}. \]

c) One cubic meter to milliliters (ml), given that 1ml = 1cm\(^3\).
\[ 1 \text{ m}^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = (10^2)^3 \text{ cm}^3 = 10^6 \text{ cm}^3 = 10^6 \text{ ml} \]

(5) 29. Convert one meter to inches, given that 1in = 2.54cm. (Round your answer to one decimal place.)
\[ 1 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{100}{2.54} \text{ in} \]

\[ = 39.4 \text{ in} \]

(5) 30. Given that \( C = \frac{5}{9}(F - 32) \) and that a thermometer reads 25° C, what is the temperature in degrees Fahrenheit.

\[ F = \frac{9}{5}C + 32 = \frac{9}{5} \cdot 25 + 32 = 45 + 32 = 77^\circ F \]