Solutions

Mathematics for Elementary School Teachers

Exam 2

April 4, 2001

The point value of each problem is given in the margin.

(9) 1. Use tests to determine whether 10101014 is divisible by the following numbers. (Show how the test works.)
   a) 6 \[1+1+1+1+4=8\] Thus 3 is not a divisor, so 6 \(\nmid\) 10101014
   b) 4 \(\not\mid\) 14
   c) 11 \[1-0+1-0+1-1-4=0\] so Yes 11 is a divisor.

(8) 2. Find the prime factorization of 500 and then make an array to illustrate all of the divisors of 500.
   \[500 = 5 \cdot 10^2 = 5 \cdot 2^2 \cdot 5^2 = 2^2 \cdot 5^3\]
   \[
   \begin{array}{c|cccc}
   & 1 & 5 & 5^2 & 5^3 \\
   \hline
   1 & 1 & 5 & 25 & 125 \\
   \hline
   2 & 2 & 10 & 50 & 250 \\
   2^2 & 4 & 20 & 100 & 500 \\
   \hline
   \end{array}
   \]
   \[7 \text{ factors}\]

(6) 3. Use the Euclidean Algorithm to find GCF(847,861).
   \[
   \begin{align*}
   &= \text{GCF}(847, 14) \\
   &= \text{GCF}(7, 14) \\
   &= 7
   \end{align*}
   \]

(5) 4. State the Fundamental Theorem of Arithmetic.
   A composite number can be expressed in one and only one way as a product of primes (except for the order of the primes.)

(5) 5. Determine whether 97 is a prime and explain how you can be sure.
   \[2, 3, 5 \text{ and } 7 \text{ are not factors so } 97 \text{ is a prime. It is not divisible by any prime } \leq \sqrt{97},\]
(5) 6. Order the fractions from smallest to largest. (Avoid calculation.)
\[
\frac{5}{9} < \frac{1}{2} < \frac{101}{99} < \frac{5}{7} < \frac{3}{3} < \frac{101}{99}
\]
\[
\frac{5}{16} \text{ larger denom. makes fraction smaller}
\]
\[
1 \text{ bigger than } 1
\]

(9) 7. Calculate and express your final answers in simplified form.

a) \( \frac{7}{15} \div \frac{21}{5} = \frac{7}{15} \cdot \frac{5}{21} = \frac{1}{9} \)

b) \( \frac{125}{7} \cdot 14 = 250 \)

c) \( \frac{7}{45} + \frac{1}{6} = \frac{7}{45} \cdot \frac{2}{2} + \frac{1}{6} \cdot \frac{15}{15} = \frac{14 + 15}{90} = \frac{29}{90} \)

(9) 8. a) Write in expanded form 200.1005 = \(2 \times 10^2 + 1 \times 10^{-1} + 5 \times 10^{-4}\)

b) Express as a decimal \(5 \cdot 10^3 + 7 \cdot 10^{-2} = 5000.07\)

c) Express in words 4.0205 = Four and two-hundred five ten-thousandths.

(5) 9. Calculate and express final answer in scientific notation.
\[
\frac{1.8 \times 10^{12}}{0.006 \times 10^4} = \frac{1.8 \times 10^{12}}{6 \times 10^{-3} \times 10^4} = 3 \times 10^{12 - 4 + 3} = 3 \times 10^1 \times 10^{11} = 3 \times 10^{12}
\]

(9) 10. Estimate. (Do not calculate exact values.)

(a) \( \frac{35013}{71} \approx \frac{35000}{70} = 500 \)

(b) 4.97 percent of 1013 ≈ \(0.05 \times 1000 = 50\)

(c) \( \frac{49}{99} + \frac{98}{102} + \frac{97}{48} \approx \frac{1}{2} + 1 + 2 = 3\frac{1}{2} \)
(9) 11. Calculate exactly
   
   a) \[36.24 - 6.007 = \frac{36.240}{30.233}\]

   b) \[.017 \times 20.1 = 0.3417\]

   c) \[2.01 \div .015 = 134\]

(5) 12. Two-fifths of a third grade class were girls. What was the ratio of boys to girls in the class? (Use whole numbers.)
   
   What percentage of the class were boys?
   
   \[\frac{3}{5} = .6 = 60\%\]

(6) 12. Convert the given decimal to a fraction in simplified form: 2.05

   \[\frac{185}{90} = \frac{37}{18}\]

(5) 13. A recipe calls for 2 cups of flour and 1/3 cup of milk. If you only have \(1\frac{2}{3}\) cups of flour, how much milk should you add? \(x = \text{amount milk}\).

   \[
   \frac{\text{milk}}{\text{flour}} \times \frac{\sqrt{3}}{2} \Rightarrow x = 1\frac{2}{3} \times \frac{1}{6} = \frac{5}{3} \times \frac{1}{6} = \frac{5}{18} \text{ cups}
   \]

   \[
   \frac{1}{2} = \frac{3}{6} \times \frac{1}{2} = \frac{1}{6}
   \]

(5) 14. The morning temperature was 40\(^\circ\). It went up 50 percent by mid-afternoon and then down 20 percent (from the mid-afternoon temp.) by early evening. What was the early evening temperature?

   \[
   40^\circ + .5 \times (40^\circ) = 40 + 20 = 60^\circ = \text{afternoon temp}
   \]

   \[
   60 - .2 \times 60 = 60 - 12 = 48^\circ = \text{evening temp}
   \]