Mathematics for Elementary School Teachers
Exam 1
February 25, 2002

The point value of each problem is given in the margin.

(8) 1. Identify each of the following sequences as arithmetic, geometric or neither and answer the questions asked.
(a) The population of a colony of fleas was 4 the first week, 8 the second week, 16 the third week, 32 the fourth week and so on. Type of sequence? Geometric. If the population continues to grow in the same manner for the next year, what is the population in the 30th week? (expressed in exponent notation) \(4, 8 = 4 \times 2^2, 16 = 4 \times 2^4, 32 = 4 \times 2^5, \ldots\)
In 30th week \(P = 4 \times 2^{29}\) or \(2^{31}\)
(b) 12, 18, 24, 30, ... Type of sequence? Arithmetic. What is the 51st number in this sequence?
\[12, 18 = 12 + 6, 24 = 12 + 2 \times 6, \ldots\]
\[51^{st} = 12 + 50 \times 6 = 12 + 300 = 312\]

(6) 2. Make Venn diagrams to illustrate
a) \(A \cap B = \) All points not in \(A\) but in \(B\).

\[\begin{array}{c}
\text{A} \\
\text{B} \\
\hline
\end{array}\]

b) \((A \cup B) - (A \cap B) = \) All points in \(A\) or \(B\) but not in the overlap of \(A \cap B\).

\[\begin{array}{c}
\text{A} \\
\text{B} \\
\hline
\end{array}\]

(8) 3. Let \(A = \{1, 3, 4, 5, 7, 9, 11\}, B = \{x | x > 5\}, C = \{2, 4, 6, 8, 10\}\).
(a) Find \(A - B = \) \[\{1, 3, 4, 5, 7, 9, 11\} \setminus \{x | x > 5\} = \{1, 3, 4, 5, 7, 9, 11\}\]
Start with \(A\) and delete points in \(B\).
(b) Find \((A \cap B) \cup C = \) \[\{1, 3, 4, 5, 7, 9, 11\} \cap \{2, 4, 6, 8, 10\} \cup \{2, 4, 6, 7, 8, 9, 10, 11\}\]
\(\cap B\)

(8) 4. a) Write the following as a base-10 numeral.
5 \cdot 10^5 + 3 \cdot 10^2 + 3 = 500303
b) Write the following base-10 numeral in expanded form.
70034050 = \(7 \times 10^7 + 3 \times 10^4 + 4 \times 10^3 + 5 \times 10\)

(6) 5. Recall the Roman numerals \(L = 50, C = 100, D = 500, M = 1000\).
(a) Convert 179 into Roman numerals.
\(CLXXIX\)
(b) Identify the year MCMLXVII.
\(1947\)
(6) 6. Express the following binary number in base-10.
\[ 1000101_{\text{two}} = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 69 \]

(6) 7. Express the base-10 numeral 100 in base-4.
Place values: 1, 4, 16, 64
\[ 100 = 1 \cdot 64 + 2 \cdot 16 + 1 \cdot 4 + 0 \cdot 1 = 1210_{\text{four}} \]

(9) 8. Fill in the blank and indicate the law illustrated. (Spelling counts!)
(a) \[ 7(x + b) = (x + \underline{b}) \cdot 7 \] **commutative**
(b) \[ 3(x + 25) = 3x + \underline{75} \] **distributive**
(c) \[ (42 + 13) + 17 = 42 + (13 + \underline{17}) \] **associative**

(9) 9. Determine whether the following sets are closed under the given operation. If not give a counterexample.
(a) \{ -1, 0, 1 \} under subtraction. **Not closed** \[ 1 - (-1) = 1 + 1 = 2 \notin S \]
(b) \{ 0, 1, 2, 3, 5, 6, 7, 8, \ldots \}, the set of whole numbers without 4, under addition.
**Not closed**, \[ 1 + 3 = 4 \notin S \]
or \[ 2 + 2 = 4 \notin S \]
(c) \{ 7, 9, 11, 13, \ldots \}, the set of odd whole numbers \( \geq 7 \), under multiplication.
**Closed. The product of two odds is odd.**

(8) 10. In a class of 40 students, 20 watched television last night, 30 did homework and 16 did both. Illustrate this information on a Venn diagram using T for students who watched television and H for those who did homework. How many students did neither one of these two activities? \[ \boxed{6} \]
(9) 11. A function \( f(x) \) is given by the set of ordered pairs
\[
\{(1, 4), (2, 7), (4, 13), (6, 19)\}.
\]
(a) What is the domain of the function \( f(x) \)?
\[
\begin{array}{c|c}
 x & y \\
\hline
 1 & 4 = 3 \cdot 1 + 1 \\
 2 & 7 = 3 \cdot 2 + 1 \\
 4 & 13 = 3 \cdot 4 + 1 \\
 6 & 19 = 3 \cdot 6 + 1 \\
 \hline
 x & 3 \cdot x + 1 
\end{array}
\]
(b) Give a formula for the function \( f(x) \). (Hint: It's of the form \( f(x) = ax + b \))
\[
f(x) = 3x + 1
\]
(c) Draw an arrow diagram for \( f(x) \).

(6) 12. A student is trying to understand why we subtract exponents when we divide two quantities
with the same base: \( \frac{b^n}{b^m} = b^{n-m} \). Explain it using a good example.
\[
\frac{b^5}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^{5-2} = b^3
\]

\[
\text{or} \hspace{1cm} \frac{7^6}{7^4} = \frac{7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7} = 7^{6-4} = 7^2
\]
you might note that there is an understood 1 in denominator after the cancellation.

(5) 13. Give a model for explaining why \( a \times b = b \times a \) for any two natural numbers \( a, b \).

Example: \( 3 \times 5 = 5 \times 3 \)

\[
\begin{array}{c}
\text{Note that this model works for any values} \\
of \ a \text{ and } b:
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
\text{Example: } 3 \times 5 = 5 \times 3 \\
\text{3 fives is the same as 5 threes.}
\end{array}
\end{array}
\]

(6) 14. Give the quotients and remainders or state that they do not exist. Also show how to write
your answers in the form \( a = qb + r \).

a) \( 0 \div 5 \)
\[
5 \overline{0.0} \hspace{1cm} q = 0 \hspace{1cm} r = 0
\]

b) \( 5 \div 0 \)

Not defined. Division by zero is not defined.

\[
6 \overline{45.3} \hspace{1cm} q = 7 \hspace{1cm} r = 3
\]

\[
45 = 7 \cdot 6 + 3
\]