The point value of each problem is given in the margin. You must show your work to receive full credit for your answers. Work neatly. Check that you have 8 pages. You have 1 hour and 50 minutes.

1. Use absolute value notation to express: The distance between \(x\) and 3 is at least 5. Then sketch the set of points on the number line provided. Indicate status of endpoints.

\[ |x - 3| \geq 5 \]

2. Simplify. No negative exponents should appear in your final answer.

\[
\left( \frac{x^{-2}y^2}{x^3} \right)^{-1} = \left( \frac{x^2}{y^2} \right)^{-1} = \left( \frac{x^5}{y^2} \right)^{1} = \frac{x^5}{y^2}
\]

3. Rationalize the denominator, and simplify.

\[
\frac{3}{\sqrt[4]{7} + 2} = \frac{3 \left( \sqrt[4]{7} - 2 \right)}{7 - 4} = \frac{3 \left( \sqrt[4]{7} - 2 \right)}{3} = \sqrt[4]{7} - 2
\]

4. Expand and express your final answer as a polynomial in standard form.

\[
(2x - 1)^3 = (2x)^3 - 3(2x)^2 + 3(2x) - 1 = 8x^3 - 12x^2 + 6x - 1
\]
(6) 5. Factor. \(4x^3 - x^2 + 12x - 3 = x^2(4x-1) + 3(4x-1)\)
\[= (4x-1)(x^2 + 3)\]

(7) 6. Perform the indicated operations and simplify.
\[
\frac{x^2 - 1}{x - 1} = \frac{x^2 - 1}{x} \cdot \frac{x}{x - 1} = \frac{(x-1)(x+1)x}{x(x-1)} = (x+1)x
\]
so \(x^2 + 3\)

(7) 7. Solve for \(x\).
\[
\left(\frac{1}{x+2} - \frac{1}{x} = \frac{2}{x}\right) x (x+2)
\]

\[
\begin{align*}
x - (x+2) &= 2 (x+2) \\
-2 &= 2x + 4 \\
2x &= -6 \\
x &= -3
\end{align*}
\]
Check: \(\frac{1}{-1} + \frac{1}{3} = -\frac{2}{3}\) OK.

(6) 8. Solve for \(s\) in terms of \(R\). \(R = \frac{2s}{s + 1}\).
\[
R(s + 1) = 2s
\]
\[
Rs + R = 2s
\]
\[
Rs - 2s = -R
\]
\[
s(R - 2) = -R
\]
\[
S = \frac{-R}{R - 2} \quad \text{or} \quad \frac{R}{2 - R}
\]
(8) 9. How many gallons of a 30\% salt solution must be added to 30 gallons of a 60\% solution to make a 40\% solution?

\[
\begin{align*}
\begin{array}{c}
x \text{ gal.} \\
30\% \\
\end{array} + \\
\begin{array}{c}
30 \\
60\% \\
\end{array} = \\
\begin{array}{c}
x + 30 \\
40\% \\
\end{array}
\end{align*}
\]

\[0.3x + 0.6 \times 30 = 0.4(x + 30)\]
\[0.3x + 18 = 0.4x + 12\]
\[6 = 0.1x\]
\[x = 60 \text{ gal.}\]

(7) 10. Solve the inequality and sketch the solution set on the number line provided.

\[5 < 1 - 2x \leq 13\]
\[5 - 1 < -2x \leq 13 - 1\]
\[4 < -2x \leq 12\]
\[-2 > x \geq -6\]

(7) 11. Find an equation of the line perpendicular to the line \(y = \frac{1}{2}x\) and passing through the point (5, -3). Put your final answer in the form \(y = mx + b\).

\[m = -\frac{2}{1} = -2\]
\[y - y_1 = m(x - x_1)\]
\[y - (-3) = -2(x - 5)\]

\[y + 3 = -2x + 10\]
\[y = -2x + 7\]
(8) 12. Rewrite the equation of the given circle in standard form and identify its center point and radius.

\[ x^2 + y^2 + 6x - 2y = 19 \]

\[
\frac{x^2 + 6x + 9}{9} + \frac{y^2 - 2y + 1}{1} = 19 + 9 + 1
\]

\[ (x + 3)^2 + (y - 1)^2 = 29 \]

center point = \((-3, 1)\)

radius = \(\sqrt{29}\)

(6) 13. Specify the domain of the function \(f(x) = \frac{\sqrt{3-x}}{x+1}\).

\[ 3 - x \geq 0 \quad \text{and} \quad x + 1 \neq 0 \]

\[ x \leq 3 \quad \text{and} \quad x \neq -1 \]

(8) 14. Sketch the graph of the function \(f(x) = \begin{cases} 
|x|, & x \leq 2 \\
4 - x, & x > 2 
\end{cases}\).
(8) 15. Solve the following equation using complex numbers if need be. Simplify your final answer(s).

\[ x^2 + 2x + 5 = 0. \]

\[ x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 5}}{2} = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i. \]

(8) 16. Sketch the graph of the quadratic function \( f(x) = -x^2 + 2x + 3 \). Identify the vertex and intercepts.

\[ \text{Vertex Point: } x = \frac{-b}{2a} = \frac{-2}{2} = 1, \quad y = -1 + 2 \cdot 1 + 3 = 4, \quad (1, 4) \]

\[ \text{x-intercept(s): } -x^2 + 2x + 3 = 0, \quad x^2 - 2x - 3 = 0, \quad (x - 3)(x + 1) = 0, \quad x = 3, -1 \]

\[ \text{y-intercept(s): } x = 0, \quad y = 3, \quad (0, 3) \]

(8) 17. Sketch the graph of \( f(x) = \frac{x^2 + 16}{2x^2 - 4} \) and identify and graph the following:

(a) Vertical Asymptote(s): \( 2x^2 - 4 = 0, \quad x^2 = 2, \quad x = \pm \sqrt{2} \)

(b) Horizontal Asymptotes(s): \( y = y_2 \)

(c) x-intercept(s): \( x^2 - 16 = 0, \quad x = \pm 4 \)

(d) y-intercept(s): \( x = 0, \quad y = \frac{-16}{0} \)

\[ \begin{array}{c|c|c|c}
\hline
x & y & y_1 & y_2 \\
\hline
1 & \frac{-12}{2} & 7 & \frac{1}{2} \\
2 & \frac{-12}{4} & -3 & \frac{1}{2} \\
\hline
\end{array} \]
(8) 18. Find the real zeros of \( f(x) = x^3 + x^2 - 8x - 12 \) with the help of your calculator and then use this information to factor the polynomial \( f(x) \) completely.

\[
\text{zeros: } x = -2, 3
\]

\[
f(x) = (x + 2)(x - 3)
\]

\[
\frac{x^2 - x - 6}{x^3 + x^2 - 8x - 12}
\]

\[
\text{even multiplicity (graph touches x-axis)}
\]

\[
\text{or consider } (x+2)(x-3)(x+3)
\]

\[
\begin{align*}
\frac{x^2 - 2x - 3}{-x^2 - 2x - 12} & \quad \frac{-x^2 - 2x}{-x^2 - 2x - 12} \\
\frac{x^2 - 2x - 3}{(x+2)(x-3)(x+3)} & \quad \frac{-x^2 - 2x}{(x+2)(x-3)(x+3)}
\end{align*}
\]

(6) 19. Write the following as the logarithm of a single quantity.

\[
\ln(2x + 1) + \ln(x) - \ln(x + 1) = \ln \left( \frac{(2x+1)x}{(x+1)} \right)
\]

(8) 20. Sketch the graph of the function \( f(x) = \log_2(x + 2) \) on the chart below and indicate the following:

\[
\begin{array}{c|c|c|c|c}
\hline
x & y & \frac{x + y}{6} & 1 & 2 \\
\hline
2 & 2 & 2 & 2 & 2 \\
\hline
\end{array}
\]

Domain of \( f(x) \):
\[
\begin{align*}
x + 2 & > 0 \\
x & > -2
\end{align*}
\]

Vertical asymptote: \( x = -2 \)

\( x \)-intercept: \( \log_2(x + 2) = 0 \) \( \Rightarrow x + 2 = 2^1 \Rightarrow x = 1 \)

(8) 21. Evaluate the following logarithms. Give the exact values in parts (a), (b) and (c) and an approximation to two decimal places in (d).

(a) \( \log_2 2 = \frac{1}{2} \)
\[
4^{1/2} = \sqrt{4} = 2
\]

(b) \( \ln(e) = 1 \)
\[
e^1 = e
\]

(c) \( \log_{10}(0.01) = -2 \)
\[
10^{-2} = \frac{1}{10^2} = .01
\]

(d) \( \log_8(10) = \frac{\ln(10)}{\ln(8)} = 1.43 \)
(6) 22. $1000 is invested in an account earning 6% interest compounded monthly. Find the balance after 10 years.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ = 1000 \left(1 + \frac{0.06}{12}\right)^{12 \cdot 10} \approx \$1819.40 \]

(8) 23. Find the time required for $500 to double if it is invested at a rate of 6% compounded continuously. Round to two decimal places.

\[ A = P \ e^{rt} \]
\[ 1000 = 500 \ e^{0.06t} \]
\[ 2 = e^{0.06t} \]
\[ \ln 2 = 0.06t \]
\[ t = \frac{\ln(2)}{0.06} \approx 11.55 \text{ years} \]

(6) 24. Write the matrix below in reduced row-echelon form, RREF. (Either show your work by hand or indicate that you have used a calculator.)

\[
\begin{bmatrix}
1 & 3 & 0 \\
2 & 7 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 0 \\
0 & 1 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & 1
\end{bmatrix}
\]

(8) 25. Solve the following system of equations by hand. You must show your work to receive credit.

\[ E_1 \quad x - y + z = 6 \]
\[ E_2 \quad -x + 2y - z = -7 \]
\[ E_3 \quad x - 2y + 2z = 10 \]
\[ E_1 + E_2 \quad y = -1 \]
\[ -E_1 + E_3 \quad -y + 2z = 4 \Rightarrow 2z = 4 + y = 3 \]
\[ x + y - 2z = 6 - 1 - 3 = 2 \]

answer(s): \((x, y, z) = (2, -1, 3)\)
(8) 26. Solve the following system of equations by hand. You must show your work to receive credit.

\[\begin{align*}
y - x^2 &= 10 \\
y - 2x &= 34
\end{align*}\]

\[
\begin{align*}
x &= 6 \\
y &= 10 + x^2 = 46
\end{align*}
\]

Subtract \(-x^2 + 2x = -2y\)

\[
\begin{align*}
x^2 - 2x - 2y &= 0 \\
(x-6)(x+4) &= 0 \\
x &= 6, -4
\end{align*}
\]

answer(s): \((x, y) = (6, 46), (-4, 26)\)

(8) 27. Solve the following system of equations on your calculator. Round to two decimal places. Draw rough graphs of the two equations to show your work.

\[\begin{align*}
y &= 3x - 4 \\
y &= 2 + e^{-x}
\end{align*}\]

Use \(\text{Int} \) and \(\text{Calc} \) to find the intersection.

answer(s): \((x, y) = (2.04, 2.13)\)

(8) 28. The following is the augmented matrix for a system of three equations in three unknowns \(x, y\) and \(z\). Write down the system of equations and then solve the system for \(x, y\) and \(z\).

\[
\begin{bmatrix}
1 & 0 & -2 & | & 0 \\
0 & 1 & 1 & | & 2 \\
0 & 0 & 1 & | & 3
\end{bmatrix}
\]

\[\begin{align*}
x - 2z &= 0 \\
y + z &= 2 \\
z &= 3
\end{align*}\]

\[\Rightarrow \begin{align*}
x &= 2z = 6 \\
y &= 2 - z = 2 - 3 = -1 \\
z &= 3
\end{align*}\]

answer: \((x, y, z) = (6, -1, 3)\)