SOLUTIONS

Rec. Instr., Time

Calculator 75-83

COLLEGE ALGEBRA, FINAL EXAM
December 18, 2002

Show all work for full credit. You may use a calculator, but do not use books or notes. The point-value of each problem is given in the left-hand margin. Read the directions carefully. You have two hours.

(6) 1. Circle all of the numbers that are rational numbers. All numbers that can be represented as a ratio of two integers. As decimals, they either have a repeating pattern or terminate.

\[ \frac{17}{1} \quad \frac{22}{10} \quad \frac{\sqrt{2}}{1} \quad 7.13 \rightarrow 7.131313... \]

(7) 2. Simplify. No negative exponents should appear in your final answer.

\[ \left( \frac{x^{-2}y^2}{y^3} \right)^{-1} = \left( \frac{1}{x^2y^{3-2}} \right)^{-1} = \left( \frac{1}{x^2y} \right)^{-1} = x^2y \]

(4) 3. a) The value of a $50,000 house went down by 20% the first year and up by 20% the second year. What was its value at the end of the second year?

After 1 year: $50,000 \times (0.8) = $40,000

After 2 years: $40,000 + (0.2) \times $40,000 = $48,000

(4) 3. b) The average SAT score increased from 500 to 532. By what percent had the average score gone up?

\[ \frac{22}{500} = \frac{44}{1000} = .044 = 4.4\% \]

(4) 3. c) The price of a television has been discounted 10%. The sale price is $540. What was the original price?

\[ x = \text{original price} \]

\[ x - (0.1)x = 540 \Rightarrow 0.9x = 540 \Rightarrow x = \frac{540}{0.9} = 600 \]

(6) 4. Expand and express your final answer as a polynomial in standard form.

\[ (2x - 1)(3x^2 - x + 2) = 2x(3x^2 - x + 2) - (3x^2 - x + 2) \]

\[ = 6x^3 - 2x^2 + 4x - 3x^2 + x - 2 = 6x^3 - 5x^2 + 5x - 2 \]
5. Factor completely the following polynomials.

(4) a) \(2x^3 - x^2 - 8x + 4 = x^2(2x-1) - 4(x-2)(x-1) = (ax-1)(x^2-4)\)
    \[= (ax-1)/(x-2)(x+2)\]

(4) b) \(x^2(2x-1) + x^2(2x-1)^2 = x^2(ax-1) \left[ x + (ax-1) \right] = x^2(ax-1)/(3x-1)\)

(7) 6. Perform the indicated operations and simplify.
\[
\frac{1-x^2}{x} + \frac{x-1}{x} = \frac{1-x^2}{x} \cdot \frac{x}{x-1} = \frac{(1-x)}{(x+1)(x-1)}
\]
\[= \frac{(x-1)(1+x)}{(x-1)(x+1)} = -(1+x) \text{ or } -1-x
\]

(7) 7. Solve for \(x\).
\[
\left[ \frac{1}{x+2} - \frac{2}{x} = 1 - \frac{x}{x} \right] \cdot x(x+2)
\]
\[
x - 2(x+2) = (x+2) \Rightarrow -6 = 2x
\]
\[
x - 2x - 4 = x + 2
\]
\[-4 = 2 = x + 2
\]
\[-4 - 2 = x + x
\]
\[x = \frac{-6}{2} = -3
\]
\[\text{check: } \frac{1}{-1} + \frac{2}{3} = -\frac{1}{3} \text{ OK}
\]

(7) 8. Solve for \(a\) in terms of \(b\).
\[b(a+b) = a\]
\[ba + b^2 = a\]
\[a - ba = b^2\]
\[a(1 - b) = b^2\]
\[\Rightarrow a = \frac{b^2}{1 - b}\]

(8) 9. How many gallons of a 20% salt solution must be added to 20 gallons of a 60% solution to make a 40% solution? (Identify any symbols you introduce.)

\[
\begin{array}{c}
\text{gal. } x \times 80\% \\
\text{? } \times 60\% \\
\text{? } \times 40\%
\end{array}
\]
\[
\begin{align*}
.2x + (60\%)\times 20 &= .4(x+20) \\
.2x + 12 &= .4x + 8 \\
.4 &= .2x \\
x &= \frac{.4}{.2} = 20 \text{ gal.}
\end{align*}
\]
(8) 10. Solve the inequality and sketch the solution set on the number line provided.
\[
\frac{x}{x+2} \leq 0
\]
Critical pts: \(x = 0, -2\)

<table>
<thead>
<tr>
<th>Test</th>
<th>(\frac{x}{x+2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{x}{x+2})</td>
</tr>
<tr>
<td>-1</td>
<td>(\frac{x}{x+2})</td>
</tr>
<tr>
<td>-3</td>
<td>(\frac{x}{x+2})</td>
</tr>
</tbody>
</table>

Excluded because \(x = -2\)
Include because \(x = 0\)

\(-2 \leq x \leq 0\)

(8) 11. Find an equation of the line perpendicular to the line \(y = -\frac{1}{2}x\) and passing through the point \((2,0)\). Put your final answer in the form \(y = mx + b\).

Given line slope: \(m = -\frac{1}{2}\)

Perpendicular slope: \(m = \frac{3}{1} = 3\)

Pt-slope form:

\[y - y_1 = m (x - x_1)\]
\[y - 0 = 3 (x - 2)\]
\[y = 3x - 6\]

(8) 12. Perform the given operations and express your answers as complex numbers in standard form \(a + bi\) with \(a, b\) real numbers. \((i = \sqrt{-1}\), the imaginary unit.\)

a) \((2 - i)(1 + 2i) = 2 - i + 4i - 2i^2 = 2 + 3i + 2 = 4 + 3i\)

b) \(\frac{5}{2 - i} \cdot \frac{2 + i}{2 + i} = \frac{5(2 + i)}{4 - i^2} = \frac{5(2 + i)}{4 - (-1)} = \frac{5(2 + i)}{5} = 2 + i\)

(8) 13. Rewrite the equation of the given circle in standard form and identify its center point and radius.

\[x^2 + y^2 + 8x - 2y = 0\]

\[x^2 + 8x + \boxed{16} + y^2 - 2y + \boxed{1} = 16 + 1\]

\[(x + 4)^2 + (y - 1)^2 = 17 = (\sqrt{17})^2\]

Center point = \((-4,1)\)

Radius = \(\sqrt{17}\)
(8) 14. Sketch the graph of the function \( f(x) = \begin{cases} |x|, & x \leq 1 \\ x - 1, & x > 1 \end{cases} \).

1) \( y = |x| \) for \( x \leq 1 \) (V-shaped graph)

2) \( y = x - 1 \) for \( x > 1 \), line with slope 1.

Open \( 0 \), because \( x > 1 \) for \( y = x - 1 \).

(8) 15. Solve the following equation using complex numbers if need be. Simplify your final answer(s).

\[ x^2 + 2x + 5 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 5}}{2} \]

\[ = \frac{-2 \pm \sqrt{-16}}{2} \]

\[ = \frac{-2 \pm 4i}{2} \]

\[ = -1 \pm 2i \]

(6) 16. Starting with the graph of \( y = \sqrt{x} \), indicate how to obtain the graphs of the following by filling in the blanks. (You might find it helpful to sketch the graphs.)

a) \( y = \sqrt{x} - 7 \).

Shift \( 7 \) units right (left, right, up or down?).

b) \( y = \sqrt{-x} + 2 \).

Reflection in the \( y \) axis, followed by

Shift \( 2 \) units up (left, right, up or down?).

(8) 17. Sketch the graph of \( f(x) = \frac{4x-4}{3+x} \) and identify and graph the following:

(a) Vertical Asymptote(s): \( x = -3 \)

(b) Horizontal Asymptote(s): \( y = \frac{4}{3} \)

(c) \( x \)-intercept(s): \( 2x - 4 = 0 \), \( x = 2 \)

(d) \( y \)-intercept(s): \( x=0 \), \( y = \frac{-4}{3} \)

\[ \text{On TI-83} \]

\[ y = (4x-4)/(3+x) \]

\[ \text{Use TI-83 TABLE} \]

\[ \begin{array}{c|c}
\hline
x & y \\
\hline
-4 & 1.09 \\
-6 & 0.33 \\
-2 & -8 \\
4 & 0.57 \\
8 & 1.09 \\
\hline
\end{array} \]
18. Let \( f(x) = x^3 + 6x^2 - 9x - 14 \).

a) According to the rational zero test, what are the possible rational zeros of \( f(x) \)?

\[ p = \text{divisor of } 14, q = \text{factor of } 1 = 1 \]

\[ \pm \frac{p}{q} = \pm \frac{1}{1}, \pm 6, \pm 7, \pm 14 \]

\[ x = -1, 2, 7 \]

b) Find the real zeros of \( f(x) \) either by using your calculator or by testing the candidates in the preceding list. Given on standard scale. Use Trace or [2: zero] to verify that \( f(x) \) has zeros at \( x = -7, 1, 2 \).

\[
\begin{align*}
\text{f}(x) &= (x+7)(x+1)(x-2) \\
\end{align*}
\]

19. Write the following as the logarithm of a single quantity.

\[
3 \ln(y) + 2 \ln(x) - \ln(z) = \ln(y^3) + \ln(x^2) - \ln(z) = \ln \left( \frac{y^3 \cdot x^2}{z} \right)
\]

20. Sketch the graph of the function \( f(x) = \log_2(x-2) \) on the chart below and indicate the following:

- Domain of \( f(x) \): \( x - 2 > 0 \), \( x > 2 \)
- Vertical asymptote: \( x = 2 \)
- \( x \)-intercept: \( x - 2 = 1 \), \( x = 3 \)
- Is there a horizontal asymptote? \( \text{NO} \)

21. Evaluate the following logarithms. Give the exact values in parts (a), (b) and an approximation to two decimal places in (d).

(a) \( \log_2 4 = \frac{2}{1} \)

(b) \( \ln(e^2) = 2 \cdot \ln(e) = 2 \)

(d) \( \log_7(14) = \frac{\ln(14)}{\ln(7)} = 1.36 \text{ year} \)

22. Find the time required for \$400 to double if it is invested at a rate of 4% compounded continuously. Round to two decimal places.

\[ \begin{align*}
A &= Pe^{rt} \\
800 &= 400 e^{0.04t} \\
2 &= e^{0.04t} \\
\ln 2 &= 0.04t \\
&= \frac{\ln(2)}{0.04} = 17.33 \text{ years}
\end{align*} \]
(8) 23. Solve the following system of equations by hand. You must show your work to receive credit.

\[ \begin{align*}
E_1 & \quad x - 3y + z = 7 \\
E_2 & \quad 2x - y - 2z = -14 \\
E_3 & \quad x - 3y = 0
\end{align*} \]

**Back substitution:**

\[ \begin{align*}
4x - 5y & = 0 \\
4x - 12z & = 0
\end{align*} \]

Subtract \( 7y \neq 0 \Rightarrow y = 0 \)

answer(s): \((x, y, z) = (0, 0, 7)\)

(8) 24. Solve the following system of equations by hand. You must show your work to receive credit.

\[ \begin{align*}
y - x = 3 \\
y - x^2 = 1
\end{align*} \]

\[ \begin{align*}
3 + x - x^2 & = 1 \\
\Rightarrow x^2 - x + 1 - 3 & = 0 \\
\Rightarrow x^2 - x - 2 & = 0 \\
\Rightarrow (x - 2)(x + 1) & = 0
\end{align*} \]

\[ \begin{align*}
x = 2 \\
x = -1
\end{align*} \]

When \( x = 2 \), \( y = 3 + x = 5 \); when \( x = -1 \), \( y = 3 + x = 2 \)

answer(s): \((x, y) = (2, 5) \text{ or } (-1, 2)\)

(6) 25. Write the matrix below in reduced row-echelon form, RREF. (Either show your work by hand or indicate that you have used a calculator.)

\[ \begin{bmatrix}
2 & 6 & 0 \\
4 & 8 & 1
\end{bmatrix} \xrightarrow{R_2 \rightarrow \frac{1}{4} R_2} \begin{bmatrix}
1 & 3 & 0 \\
1 & 2 & 1
\end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{bmatrix}
1 & 3 & 0 \\
0 & -1 & 1
\end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + 1} \begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 1
\end{bmatrix} \]

In RREF each row must have a leading 1, the leading 1’s progress to the right as you move down and in any column with a leading 1, all other entries are 0.

(8) 26. The following is the augmented matrix for a system of three equations in three unknowns \( x, y, \) and \( z \). Write down the system of equations and then solve the system for \( x, y, \) and \( z \).

\[ \begin{bmatrix}
1 & 5 & -2 & | & 0 \\
0 & 1 & 3 & | & 2 \\
0 & 0 & 1 & | & 0
\end{bmatrix} \xrightarrow{x + 5y - 2z = 0} \begin{bmatrix}
1 & 3 & 0 & | & -2 \\
0 & 1 & 3 & | & 2 \\
0 & 0 & 1 & | & 0
\end{bmatrix} \xrightarrow{z = 0} \begin{bmatrix}
1 & 3 & 0 & | & 2 \\
0 & 1 & 3 & | & 2 \\
0 & 0 & 1 & | & 0
\end{bmatrix} \xrightarrow{\text{From 1st equation,}} \begin{bmatrix}
1 & 0 & 3 & | & 6 \\
0 & 1 & 3 & | & 2 \\
0 & 0 & 1 & | & 0
\end{bmatrix} \xrightarrow{\text{From 2nd equation,}} \begin{bmatrix}
1 & 0 & 0 & | & -10 \\
0 & 1 & 0 & | & 0 \\
0 & 0 & 1 & | & 0
\end{bmatrix} \]

answer(s): \((x, y, z) = (-10, 0, 0)\)