Show all work for full credit. You may use a calculator, but do not use books or notes. The point value of each problem is given in the left-hand margin. Read the directions carefully. You have 60 minutes.

(8) 1. Find the midpoint of the line segment joining \((3, 2), (0, -6)\) and find the distance between the two points \((3, 2), (0, -6)\).

\[
\text{midpoint } = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + 0}{2}, \frac{2 + (-6)}{2} \right) = \left( \frac{3}{2}, \frac{-4}{2} \right) = \left( \frac{3}{2}, -2 \right)
\]

\[
\text{distance } = \sqrt{\left( x_1 - x_2 \right)^2 + \left( y_1 - y_2 \right)^2} = \sqrt{\left( 3 - 0 \right)^2 + \left( 2 - (-6) \right)^2} = \sqrt{9 + 64} = \sqrt{73}
\]

(8) 2. Rewrite the equation of the given circle in standard form and identify its center point and radius.

\[
x^2 + y^2 + 2x - 6y = 15
\]

\[
(x - h)^2 + (y - k)^2 = r^2
\]

\[
x^2 + 2x + \frac{1}{4} + y^2 - 6y + \frac{9}{4} = 15 + \frac{1}{4} + \frac{9}{4}
\]

\[
(x + 1)^2 + (y - 3)^2 = 25 = 5^2
\]

center point = \((-1, 3)\)

radius = 5

(7) 3. Solve for \(A\) in terms of \(B\) : \(\frac{3}{A} = \frac{B}{A+1}\)

\[
3(A+1) = B \cdot A
\]

\[
3A + 3 = BA
\]

\[
3A - BA = -3
\]

\[
A(3-B) = -3
\]

\[
A = \frac{-3}{3-B} \quad \text{or} \quad A = \frac{3}{B-3}
\]
(7) 4. Use your calculator to make an accurate sketch of the graph of \( y = .13x^3 - 6x + 6 \). Determine window settings from the axes labelled below and answer the questions below.

\[
\begin{align*}
\text{Window:} \\
X_{\text{min}} &= -20 \\
X_{\text{max}} &= 20 \\
X_{\text{sc}} &= 2 \\
Y_{\text{min}} &= -50 \\
Y_{\text{max}} &= 100 \\
Y_{\text{sc}} &= 10
\end{align*}
\]

What is the \( y \)-intercept? \( x = 0, y = 6 \)

What is \( y \) when \( x = 5 \)? \( y = -7.75 \)

(12) 5. a) The value of a $50,000 house went up by 10% the first year and down by 10% the second year. What was its value at the end of the second year?

After 1 year: \( 50000 \cdot (1.1) \cdot 50000 = $55000 \)

After 2 years: \( 55000 \cdot (1) \cdot 55000 = $49,500 \)

b) A $20,000 new car was worth only $16,000 one year later. By what percent had the value gone down?

\[
\frac{4000}{20000} = \frac{1}{5} = .2 = 20\% 
\]

c) The price of a television has been discounted 15%. The sale price is $400. What was the original price?

Let \( x = \text{original price} \)

\[
x - .15x = 400 \Rightarrow .85x = 400 \Rightarrow x = \frac{400}{.85} = $470.59 
\]

(7) 6. Complete the table below and sketch the graph of \( y = |x - 3| \). Indicate the \( x \) and \( y \) intercepts of the graph. (You can do this one by hand or on your calculator.)

| \( x \) | \( y = |x - 3| \) |
|-------|-----------------|
| 0     | 3               |
| 3     | 0               |
| 6     | 3               |
| -5    | 8               |

\( x \)-intercept: Set \( y = 0 \)

\( |x - 3| = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3 \)

\( y \)-intercept: Set \( x = 0 \)

\( y = |6 - 3| = 3 \)
(8) 7. How many gallons of a 20% boric acid solution must be added to 3 gallons of a 50% solution to make a 40% solution?

Let \( x \) gallons of 20% solution.

\[
\begin{align*}
\text{Vol.} & \rightarrow \begin{array}{c} x \\ 20\% \end{array} + \begin{array}{c} 3 \\ 50\% \end{array} = \begin{array}{c} x + 3 \\ 40\% \end{array} \\
\text{%} & \rightarrow \begin{array}{c} 20 \times \end{array} + \begin{array}{c} 50 \times 3 = 40 (x+3) \\
-20x & \begin{array}{c} 20x + 150 = 40x + 120 \\
-120 & \begin{array}{c} 150 = 20x + 120 \\
-120 & \begin{array}{c} 30 = 20x \\
\end{array}
\end{array}
\end{array}
\end{align*}
\]

\[x = \frac{30}{20} = \frac{3}{2} = 1.5 \text{ gal}\]

(8) 8. Find all real solutions \( x \) of the following equation. (Show all work by hand. You can check your answer on your calculator.)

\[x^4 + x^3 - 8x - 8 = 0\]

\[x^3(x+1) - 8(x+1) = 0\]

\[\begin{align*}
(x+1)(x^3 - 8) & = 0 \\
x+1 & = 0 \quad \text{or} \quad x^3 - 8 = 0 \\
x & = -1 \quad \text{or} \quad x^3 = 8, x = \sqrt[3]{8} = 2
\end{align*}\]

\[x = -1, 2\]

(8) 9. Find all real solutions \( x \) of the following equation. (Show all work by hand.)

\[x - \sqrt{x} = 6\]

\[\begin{align*}
(x-6)^2 & = (\sqrt{x})^2 \\
(x-6)(x-6) & = x \\
x^2 - 12x + 36 & = x \\
x^2 - 13x + 36 & = 0 \\
(x-9)(x-4) & = 0
\end{align*}\]

\[\begin{align*}
\text{check:} & \quad x = 9, \quad 9 - \sqrt{9} = 6 \quad \text{OK} \\
& \quad x = 4, \quad 4 - \sqrt{4} = 6 \quad \text{NO!}
\end{align*}\]

\[x = 9\]
(8) 10. Solve for \( x \).
\[
\left[ \frac{4}{x} - \frac{1}{x-3} = \frac{3}{x(x-3)} \right] x(x-3)
\]

\[4(x-3) - x = 3\]
\[4x - 12 - x = 3\]
\[3x = 15\]
\[x = \frac{15}{3} = 5\]

Check:
\[\frac{4}{5} - \frac{1}{2} = \frac{3}{5} \cdot 2\]
\[\frac{8}{10} - \frac{5}{10} = \frac{3}{10} \quad \text{OK.}\]
\[X = 5\]

(8) 11. Solve the following equation by hand, any way you like. Express your final answer(s) in simplified form with a radical sign. (Do not express your answer(s) in decimal form.)
\[x^2 - 4x = 6\]

\[x^2 - 4x - 6 = 0\]

Quad. Formula:
\[x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-6)}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 + 24}}{2} = \frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = \frac{4}{2} \pm \frac{2\sqrt{10}}{2} = 2 \pm \sqrt{10}\]

(12) 12. Perform the given operations and express your answers as complex numbers in standard form \( a + bi \) with \( a, b \) real numbers. \((i = \sqrt{-1}, \text{the imaginary unit.})\)

a) \((3 - i)(6 + 2i) = 18 + 6i - 6i - 2i^2 = 18 - 2(-1) = 18 + 2 = 20\]
\[i^2 = -1\]

b) \(\frac{10}{2 - i} = \frac{10}{2 - i} \cdot \frac{2 + i}{2 + i} = \frac{10(2 + i)}{4 - (-1)} = \frac{10(2 + i)}{5} = \frac{2}{2 + i}\]

\[Z(2 + i) = \frac{10}{5} \cdot \frac{2 + i}{1} = \frac{4 + 2i}{1} = 4 + 2i\]

C) \(i^n + \sqrt{-9} = i^4 \cdot i + \sqrt{9(-1)} = i + 3i = 4i\)
\[i^4 = 1\]