Studio 5 – September 29, 2006
Describe the Squirt

We have fit data to lines. Unfortunately not all data sets fit well to lines, as we have seen with population data.

Open your web browser and go to the URL:


by clicking on the link “Describe the Squirt” on the main course website.

You will see a squirt; that is, a snap-shot of water flowing from a fountain. We are going to describe the shape of the water’s flow. The shape resembles the graph of a quadratic, and so we will try to model it with such.

PROBLEM 1

We have learned that a quadratic function is determined by its vertex \((h,k)\) and another distinct point on the graph. Together they determine the equation \( f(x) = a(x-h)^2 + k \). To model the shape of the water with this equation, we first need data describing its shape.

To collect the data we need to plot points.

1. Click on the black button to clear the points that may be already marked.
2. Click on the image where you think its vertex is located, and copy the coordinates by hand.
3. Click on the blue button to mark the point.
4. Repeat steps 2 and 3 to mark a second distinct point.

Now solve for the number \(a\), as you have done on the online homework. You can do this by hand, or using Excel.

Enter the corresponding quadratic function into the website. Remember to use * for multiplication. Also note that you may enter the formula in any form you wish, not only standard form.

Then enter the lower and the upper bounds for \(x\). To determine this, click on the left most part of the squirt and read off the \(x\) value and then click on the right most part of the squirt and read off the \(x\)
value. Hit ENTER, or click on the button labeled “Try it!!”

**How close is the model to the actual shape?** Try a different second point, further away from the vertex, if it is not a close fit; or try a new guess for the vertex. Also, you may have rounded too small; that is, left off significant digits. Be sure to have 3 significant digits (non-zero digits after the decimal point, in this case) for the value of \( a \).

Note, if you modify the formula and try again, it will modify the image in a new window. However, it may not bring this new window to the foreground; which means you may have to bring the window to the front.

For this problem you are to turn in:

1. The first formula you derived and the coordinates of the vertex and the second point.
2. The second formula you derived and the new coordinates of vertex and/or second point.
3. A description of how the shape changed as you changed points and an explanation of why?

**PROBLEM 2**

We now will try to fit the equation \( f(x) = ax^2 + bx + c \). We will need at least three points this time to find the coefficients \( a, b, \) and \( c \). However, we want a larger data set so we may compute the residuals of the set. Moreover, the larger the data set the better the fit will be for the model.

So, as before, mark points on the squirt. But this time do so from the far left all the way to the right, getting 15 points.

1. Click on the black button to clear the points that may be already marked.
2. Click on the image at a point on the far left.
3. Click on the blue button to mark the point.
4. Repeat steps 2 and 3 to mark fifteen distinct points on the image of the water.
5. Click on the red button to list the coordinates of the points you have marked.

A “Java Applet Window” will open. Scroll down and you will see an array of points. The numbers in the first column are the \( x \)-coordinates and the numbers in the second column are the \( y \)-coordinates. Highlight and copy these numbers. To copy the number type CTRL-C after they are highlighted.

Open a new Excel sheet, and paste the data set into the sheet so the \( x \) values start at cell A2 and the \( y \) values start at cell C2. Label the first column \( x \) and the third one \( y \). Then make another column for \( x^2 \) that will start at cell B2. Then enter \( =A2^2 \) in the cell B2, highlight the cell, and drag the little black box down to B16 (remember you chose 15 points).
Now we will compute the coefficients. Highlight cells G2, H2, and I2 and type =LINEST(C3:C16;A3:B16). Then hit CTRL+SHIFT+ENTER. This will compute the coefficients. The number in cell G2 will be the number \(a\), and the number in H2 will be \(b\), and the number in I2 will be \(c\).

Now enter in the formula for the quadratic into the website and see how well it fits. Remember to make sure you have enough decimal places for the coefficients \(a\), \(b\), and \(c\). **Why do you think this would matter?** Also remember to enter in the largest and smallest values of \(x\) into the website, and to use the * key for multiplication.

Note: to get Excel to show more decimal places in a cell, highlight the cell and click on the button “Number Format: Add Decimal Place” on the menu bar.

For this problem you are to turn in:

1. The computed values of \(a\), \(b\) and \(c\); and the formula you entered.
2. A description of why you think the number of decimal places included in the coefficients would make a difference in whether the graph was a good fit.

PROBLEM 3

Next, we wish to compute the predicted values of the model. We will call these “\(\hat{y}\)” and denote them by \(\hat{y}\). Starting in cell D2 of the Excel sheet you were working with in problem 2, type =G$2*B2+H$2*A2+I$2 and hit ENTER. Highlight the cell and drag the little black box down to D16. These are the predicted values by our model.

Next, graph the two data sets together, the \((x,y)\) points and the \((x, \hat{y})\) points, and compare them. Then, compute the residuals \((y- \hat{y})\) in column E. **Does the model predict the behavior well?** Explain why you believe the model is good, or bad.

For this problem you are to turn in:

1. The residuals of the data.
2. A description of how the two data sets compare to each other.
3. An explanation of whether the model was good, or bad. You must say why you think your answer is correct.

EXTRA CREDIT

We have solved two linear equations in two unknowns. Geometrically each such equation determines a line, and so solving the simultaneous equations amounts to determining where two lines intersect, if
at all.

Now given three points on an unknown quadratic curve \( (x_1, y_1), (x_2, y_2), \) and \( (x_3, y_3) \); we get three linear equations in the three unknowns \( a, b, \) and \( c \) by substituting the known \( x \) and \( y \) values from the three points into the quadratic equation \( y = ax^2 + bx + c \).

Solving these equations amounts to finding where three planes intersect, if at all. In this way we can find the coefficients \( a, b, \) and \( c \).

**For extra credit, solve this system of equations.**

In a like manner, we take three points and substitute each into the expression \( y = a(x - h)^2 + k \) to again yield three equations in three unknowns. But this time the unknowns are \( a, h, \) and \( k, \) and the equations are NOT linear in the unknowns; and hence much more complicated.

**For extra credit, write out this system and compare it to the one you just solved.**