Analytic Geometry and Calculus II
Exam III
April 21, 1998

Show all work for full credit. You may use a calculator, but no books or notes.

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Some formulas for conic sections:

\[ a = \frac{c}{e}; \quad a^2 = b^2 + c^2; \quad c^2 = a^2 + b^2; \quad \text{directrix } \frac{c}{e^2}; \quad x^2 = 4py; \quad \frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1 \]

(10) 1. Graph the conic section \[ x^2 + 2x - y^2 + 2y + 4 = 0. \]
(10) 2. Show that the curve \( r = \frac{\cos \theta}{\sin^2(\theta)} \) is a parabola, and find its focus.

(10) 3. If the asymptotes of a hyperbola are \( y = \pm x \), find its eccentricity.

(10) 4. Find an equation of the ellipse with eccentricity .6 and foci (1,2) and (1,4).
5. Sketch a graph of $r = 1 - 4\sin^2(\theta)$. Label the angles where $r = 0$.

6. Find the area inside $r^2 = 4\sin(2\theta)$. 
(10) 7. If the series converges, compute its sum. Otherwise, explain why it diverges.

(a) \( \sum_{n=1}^{\infty} \left( \frac{e}{\pi} \right)^n \)

(b) \( \sum_{n=1}^{\infty} \frac{2^n - 1}{2^n} \)

(10) 8. Find the degree three Taylor polynomial \( P_3(x) \) at \( a = 0 \) for the function \( f(x) = x \cos(2x) \).
(10) 9. Show that the sequence \( a_n = \left(1 - \frac{3}{n}\right)^n \) converges.

(10) 10. Peter, Paul, and Mary take turns tossing a coin until one of them wins by tossing the first “head”. Calculate for each person the probability that they win the game.