ANALYTIC GEOMETRY AND CALCULUS II
Second Exam
October 18, 1994

The point value of each problem is indicated in the left margin. You must show all of your work for full credit. Points will be deducted for faulty reasoning and for sloppy notation. You may not use a calculator, your class notes, or any reference material. Be neat and clear. **You do not need to simplify your answers unless you are asked to do so.**

1. Integrate:

   (8) (a) \( \int \sin^2 x \cos^3 x \, dx \)

   (8) (b) \( \int \frac{dx}{x(9 + x^2)} \)

   (8) (c) \( \int_1^3 \frac{\sqrt{9 - x^2}}{x^2} \, dx \)
(8) (d) \[ \int \frac{-5xdx}{(x + 2)(x^2 + 1)} \]

(8) (e) \[ \int x\sqrt{x^4 + 1} \, dx \]

2. Consider the integral \[ \int_2^8 \sqrt{1 + \sqrt{x}} \, dx \] let \( n = 6 \). Write the formula to estimate the integral by

(8) (a) The Trapezoidal Rule

(8) (b) Simpson’s Rule

Just write the formulas. Don’t compute the values.
3. Evaluate each of the following improper integrals, or show that it diverges.

(8) (a) \[ \int_{1}^{2} \frac{dx}{2x - 3} \]

(8) (b) \[ \int_{0}^{\infty} xe^{-x} \, dx \]

4. Evaluate the limits.

(8) (a) \[ \lim_{x \to 0^+} \frac{x^3}{x - \sin x} \]

(8) (b) \[ \lim_{x \to 0} (1 - x^2)^{1/x} \]
5. Write the fourth-order Taylor polynomial \((n = 4)\) for

\[ f(x) = \ln x \quad \text{at} \quad a = 1. \]

Also write the Lagrange form of the remainder.

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(6) EXTRA CREDIT: Show that

\[ \left| \frac{1 - \cos x}{x} \right| \leq \frac{|x|}{2} \quad \text{for all} \quad x \neq 0. \]