1. (25pts) Integrate:
   a. \[ \int \frac{x + 1}{x^2 + x - 2} \, dx = \]
   b. \[ \int_{0}^{\infty} \frac{2}{3 + 4x^2} \, dt = \]
   c. \[ \int_{0}^{\pi/3} \sin^3(y) \, dy = \]
2. (25pts) In studying the spread of an epidemic, let \( y(t) \) be the fraction of the population having the disease at time \( t \) (months). At time \( t = 0 \) suppose that 2% of the population is sick, and suppose that after a month the rate of spread of the disease \( (dy/dt) \) starts to slow down. We assume that \( y \) satisfies the logistic equation \( dy/dt = ky(1 - y) \) where \( k \) is to be determined. (a) Integrate this equation,

(b) find the constant of integration,
(c) solve for $y$, 

(d) Find $k$. 
3. (25pts) Let \( f(x) = Cxe^{-2x} \) be a probability density on the interval \([0, \infty)\).

a. Compute the expectation \( E(X) = \int_0^\infty xf(x)dx \) in terms of \( C \).

b. Compute the distribution function \( F(t) = P(X \leq t) = \int_0^t f(x)dx = 1/2 \), in terms of \( C \).

c. Find \( C \) so that \( f(x) \) is a probability density on \([0, +\infty)\).
4. (25pts) A tank originally contains 100 gal of fresh water. Water with 0.5 lb/gal of salt is poured into the tank at a rate of 2 gal/min, and the well-stirred mixture is allowed to leave at the same rate. (a) Set up a differential equation for the amount of salt in the tank at time $t$ and integrate it.
(b) After 10 minutes the water flow is shut-down so that the rates \( r_{in}(t) = r_{out}(t) = r(t) \) stay equal and decrease linearly from 2 gal/min to 0 gal/min in 5 minutes. Find a formula for \( r(t) \), set up a new differential equation for the amount of salt in the tank at time \( t \) and solve it in the interval \( 10 \leq t \leq 15 \).