1. (20pts) Differentiate the following functions:
   a. \( f(x) = \sqrt{x} \ln x \)

   b. \( y = (\ln x)^x \)

   c. \( g(t) = e^{3+\ln t} \)

   d. \( f(x) = \arctan(e^x) \)
2. (20pts) Integrate:

a. \( \int_{2}^{3} \frac{dx}{x \ln^2 x} = \)

b. \( \int_{0}^{2\pi} \frac{\sin \theta}{2 - \cos \theta} d\theta = \)

c. \( \int_{0}^{1} \frac{1}{10^t} dt = \)

d. \( \int \frac{x^4}{1 + x^{10}} dx = \)
3. (24pts) Let \( f(x) = \frac{x^3}{4 \cdot 10^4} \ln \frac{x}{3 \cdot 10^2} \), for \( x > 0 \).

a. (6pts) Find the following limits:
\[
\lim_{x \to 0^+} f(x) =
\]

\[
\lim_{x \to +\infty} f(x) =
\]

b. (5pts) \( f'(x) = 0 \) has one solution. Find it.

c. (5pts) \( f''(x) = 0 \) has one solution. Find it.
d. (2pts) How does $f(x)$ behave for $x$ small? In particular, what is the sign of $f(x)$ for $x$ small?

e. (6pts) Graph $y = f(x)$, labeling the extremum and the inflection point. (Hint: to evaluate $f(x)$ at the critical point and at the inflection point use the following approximate values: $e^{-1} = .36, \quad e^{-1/3} = .72, \quad e^{-5/6} = .43, \quad e^{-5/2} = .08$)
a.(10pts) Suppose that interest on money in the bank accumulates at an annual rate of 7% compounded continuously. How much money should be invested today so that 20 years from now it will be worth $20000? (Hint: use $e^{1.4} = 4$)

b.(6pts) One of the guiding principles of most sports is “keep your eye on the ball”. In baseball, a batter stands 2 feet from home plate as a pitch is thrown with a velocity of 130 ft/sec (about 99 mph). At what rate does the batter’s angle of gaze change when the ball crosses home plate? (Aside: humans can accurately track objects only at the rate of 3 radians/second.)
5. (20pts) Suppose that a room containing 1000 cubic feet of air is originally free of carbon monoxide. Beginning at time \( t = 0 \) cigarette smoke containing 4 percent carbon monoxide is introduced into the room at a rate of 0.1 ft\(^3\)/min, and the well-circulated mixture is allowed to leave the room at the same rate.

a. (10pts) Find an expression for the amount \( y(t) \) of carbon monoxide (in cubic feet) in the room at time \( t \), by solving a differential equation of the form \( \frac{dy}{dt} = C_{in} \cdot r - C_{out} \cdot r \), where \( C \) stands for concentration and \( r \) stands for rate (you’ll need to figure out \( C_{in} \), \( C_{out} \), and \( r \) from the word problem first).

b. (10pts) Extended exposure to a carbon monoxide concentration as low as 0.012% is harmful to the human body. Find the time at which this concentration is reached. (Use the fact that \( \ln(1 + x) \) is approximately equal to \( x \) when \( x \) is small.)
6. (10 Extra pts) You drop a rock (with zero initial velocity) from a bridge. Take $g = 10 \text{m/sec}^2$. Because of air resistance (which we assume to be proportional to the velocity $v(t)$), the rock would eventually approach a downward terminal velocity of 100 m/sec if it fell indefinitely through the air. However, it does not fall indefinitely: after exactly 10 seconds it hits the water. How fast is the rock falling when it hits the water? (use $e^{-1} = .36$)