Math 221 - Review for Final Exam

- Exponential and Logarithmic functions: definitions, properties, derivative and integration.
  a) Simplify \( \log_3 \frac{1}{9} \)  
  b) \( y = \log_4(\cos x) \) find \( y' \)

- Implicit differentiation; derivatives of inverse functions
  \( xe^y = y \)

- Logarithmic differentiation: \( y = \sqrt{\frac{x+1}{x-1}} \)

- Natural growth and decay, half-life.
  \( P_0 = 2 \) mil, rate of growth 20\% find doubling time.

- Linear first order differential equations: separation of variables.
  Solve \( P' = 20P - 10, P(0) = 10 \).

- Inverse trigonometric functions; derivatives
  Use implicit differentiation to find \( \sin^{-1}(x) \)

- Indeterminate forms and L’Hopital’s rule
  a) Find \( \lim_{x \to -\infty} (1 + \frac{1}{x})^x \)  
  b) \( \lim_{x \to 0} \frac{e^{x}-1}{\sin x} \)

- Hyperbolic functions and inverse hyperbolic functions
  Use the definition of \( \sinh x \) to solve for \( x \) in \( 2 = \sinh x \)

- Integration methods: substitutions, trigonometric substitutions and integration by parts. Integrate:
  a) \( \int e^{-\frac{3}{x^2}} \, dx \)  
  b) \( \int \frac{1}{\sqrt{1-x^2}} \, dx \)  
  c) \( \int \sin x \cos^2 x \, dx \)
  d) \( \int \sqrt{x} \ln x \, dx \)  
  e) \( \int \arcsin x \, dx \)  
  f) \( \int x^2 e^x \, dx \)  
  g) \( \int \sin x \, e^x \, dx \)

Integration

9.5 - Rational functions and Partial Fraction Decomposition

9.6 - Trigonometric substitutions \( a^2 - u^2 \sin x, \ u^2 - a^2 \frac{1}{\sin x}, \ a^2 + u^2 \tan x \)

9.7 - Integrals containing quadratic polynomials (complete the square and use a trigonometric (or hyperbolic) substitution (or use the formula giving some inverse trigonometric function).

9.8 Improper integrals: defined as limits of definite integrals. The limit point is +/- infinity or a discontinuity of the integrand.

Conic Sections

Locus: The set of points \( P \) such that \( PF = e \cdot PL \) (\( F \) focus, \( L \) directrix, \( e \) eccentricity).

10.4 - Parabola; \( y^2 = 4px \), \( V(0,0) \), Focus(\( p, 0 \)), Directrix \( x = -p \)

10.5 - Ellipse; \( \frac{x^2}{a^2} + \frac{y^2}{b^2} \) Center(\( 0,0 \)), Foci(\( +/−c, 0 \)), \( c^2 = a^2 - b^2 \) \( 0 < b < a \).

10.6 - Hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} \) Center(\( 0,0 \)), Foci(\( +/−c, 0 \)), \( c^2 = a^2 + b^2 \) \( 0 < b < a \), asymptotes \( y = \pm \frac{b}{a}x \)

10.7 - Equations for Rotation of Axes and Classification of Conics (discriminant \( B^2 - 4AC \))

10.2 - Polar Coordinates \( x = r \cos \theta (or \cos \theta = \frac{adj}{hyp.}) \)  
  \( y = r \sin \theta (or \sin \theta = \frac{opp.}{hyp.}) \)

Polar curves \( < → \) Cartesian curves, intersecting polar curves.

10.3 - Area computations in polar coordinates. \( dA = \frac{1}{2} r^2 d\theta \). Area between two curves.

Infinite Series

11.2 - Infinite Sequences

Limit laws (+/-, .:/), Squeeze Law, Bounded monotonic Sequence.

Recursive relations and sequences.
Problems

1) $\int \frac{x^3 + x + 1}{x(x^2 + 1)} \, dx$

2) $\int \frac{1}{4x^2 + 4x + 5} \, dx$

3) $\int x\sqrt{1 - x^2} \, dx$

4) $\int \frac{1}{(x^2 + 1)^{3/2}} \, dx$

5) Find the area under the curve $y = \frac{1}{x^2}$ from 1 to infinity.

6) Determine if the improper integral for the curve $y = \frac{1}{\sqrt{x}}$ from -1 to 3 is convergent. Find the volume of the solid obtained by rotating the curve about the x-axis.

7) Find the equation to describe all the points $P(x,y)$ which are equidistant from $(3,2)$ and $(7,4)$.

8) Draw the graph of the polar equation $r = 2 + 3\cos \theta$.

9) Find the area outside of $r=2$ and inside $r = 4\cos \theta$.

10) Sketch the following conic section (label the foci, vertices, directrices, axes, center): $x^2 - 4y - 4x = 0$

11) Sketch the graph of $\frac{(y-2)^2}{4} - \frac{(x+1)^2}{9} = 1$

12) Determine if the following sequences converge: a) $a_n = 3^n \frac{n}{n!}$, b) $a_n = \frac{n^2}{e^n}$ c) $a_n = (\frac{e}{10})^n$

d) $a_n = \frac{n^6}{e^n}$

13) Determine if the sequence $a_{n+1} = \sqrt{a_n + 2}, a_1 = 2$ converges and if so, compute its limit.

- Geometric series, telescopic sums, p-series, harmonic series.

- Determine whether the infinite series converges or diverges. If it converges find its sum:
  a) $\sum_{n=0}^{\infty} 42^{-n}$
  b) $\sum_{n=0}^{\infty} \frac{3}{n^2 + 2n}$

- $N^{th}$ term test for divergence, integral test and remainder estimate, comparison test, limit comparison test.

- Determine whether the infinite series converges or diverges. If it converges find its sum:
  a) $\sum_{n=0}^{\infty} \frac{\sqrt{\pi}}{n+5}$
  b) $\sum_{n=0}^{\infty} \frac{\sin(\frac{x}{n})}{7n}$
  c) $\sum_{n=0}^{\infty} \frac{n!}{2^n}$

- Taylor’s formula, series and polynomial.

  Find the 4th degree Taylor formula for $f(x) = \cos x$ at 0.

- Power series, radius and interval of convergence, term-wise differentiation and integration, algebra of power series.

  1) Write the power series representation for $\frac{1}{1+x}$, then term-wise integrate to find the Taylor series of $\ln x$ at 0. Use Taylor’s formula to estimate the remainder and finally compute $\ln(1.1)$ accurate to three decimal places.

  2) Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{x^n}{n}$. 