ANALYTIC GEOMETRY AND CALCULUS II
Final Exam
May 10, 1994

The point value of each problem is indicated in the left margin. You must show all of your work for full credit. Points will be deducted for faulty reasoning, for sloppy notation, and for failure to simplify answers, even if your answer is correct. You may use a calculator, your class notes, and any reference material. Explicitly cite, in some manner, any published formulae you use.

(7) 1. Find $\frac{d}{dx} \tan^{-1}(e^{x^2})$.

Answer

(7) 2. Find $\int_0^\pi \frac{\sin \theta}{2 - \cos \theta} d\theta$.

Answer
3. Assume that \( f \) is invertible and differentiable everywhere with \( f(5) = 7 \) and \( f'(5) = -2 \). What is \((f^{-1})'(7)\)?

Answer

4. Find \( \int \frac{dx}{9 + 4x^2} \).

Answer \(-C\)
5. Find \( \int x \sin 2x \, dx \).

Answer \( \quad \) – \( C \)

6. Find constants \( A \), \( B \) and \( C \) so that

\[
\frac{1}{x(x^2 + x + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + x + 1}.
\]

Answer \( A = \quad \), \( B = \quad \), \( C = \quad \)
7. Use the Trapezoidal Rule with \( n = 4 \) to approximate
\[
\int_0^1 e^{-x^2} \, dx.
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( e^{-x^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>1.0000000</td>
</tr>
<tr>
<td>0.125</td>
<td>0.9844942</td>
</tr>
<tr>
<td>0.250</td>
<td>0.9394170</td>
</tr>
<tr>
<td>0.375</td>
<td>0.8688173</td>
</tr>
<tr>
<td>0.500</td>
<td>0.7787980</td>
</tr>
<tr>
<td>0.625</td>
<td>0.6766358</td>
</tr>
<tr>
<td>0.750</td>
<td>0.5697843</td>
</tr>
<tr>
<td>0.875</td>
<td>0.4650427</td>
</tr>
<tr>
<td>1.000</td>
<td>0.3678795</td>
</tr>
</tbody>
</table>

Answer: 

8. Find \( \lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} \).

Answer: 

(7) 9. Show that \( a_k = \sqrt{2^{k+1}} \) is a decreasing sequence.

(7) 10. Determine whether \( \sum_{k=1}^{\infty} (1 + \frac{1}{k})^k \) converges.

Answer

\[ \]
(7) 11. Find the 4th order Taylor polynomial based at $a = \frac{\pi}{2}$ for $\cos x$.

Answer $P_{4, \frac{\pi}{2}}(x) =$ 

(7) 12. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{3^k}{k^2} x^k$.

Answer 

(8) 13. Consider the equation \( r = 2 + \cos \theta \).

(a) Sketch its polar graph in the \( xy \)-plane.

(b) Sketch its complete pre-image in the \( \theta r \)-plane.
(8) 14. Find the area of the region bounded by the polar graphs $\theta = 0$, $r = \theta$ and $r = \frac{1}{2}(\theta + \pi)$.

Answer: ____________________________