Work as many of the following problems as you can in one hour. To assure partial credit, **show your work**. No calculators are permitted for this exam.

(10) 1. Sketch the graphs of \( e^{-1} \), \( \ln x \), and \( \ln(-x) \) in the space provided below, all together. (Try to display the essential features of these functions.)
2. An archaeologist has found an ancient Greek clay pot, containing 70% as much $^{14}C$ as a recently manufactured, comparable pot. The half life of $^{14}C$ is 5730 years. How old is the pot? (Express your answer in terms of $\ln(0.7)$ and $\ln(0.5)$.)

3. Evaluate each of the following indefinite integrals.

   (a) $\int \frac{6dx}{x(x-1)(x+2)}$

   (b) $\int xe^{-x}dx$

   (c) $\int \sec^2 u \tan^2 u du$
4. This problem concerns the indefinite integral \( \int \frac{x^3 \, dx}{\sqrt{x^2 - 1}}. \)

(8) (a) Use trigonometric substitution to set up the problem as an integral of a trigonometric function. (Big hint: you should wind up having to integrate a power of \( \sec \theta \).)

(8) (b) Perform the integration, and finish in the usual way, arriving at a function of \( x \).
5. Evaluate each of the following limits. That is, in each case, determine whether the limit is a real number, and if it is, determine the number.

(a) \( \lim_{x \to \infty} \frac{2x^3 + 7x^2 + \sqrt{x}}{3x^3 + 3\sqrt{x}} \)

(b) \( \lim_{n \to \infty} \frac{(\ln(n))^2}{n} \)

(c) \( \lim_{x \to 0^+} (1 - 2x)^{1/x} \)

(d) \( \lim_{n \to \infty} (1 + 0.3 + (0.3)^2 + \cdots + (0.3)^n) \)

6. Find the volume of the solid obtained by revolving the graph of \( e^{-2x} \) around the \( x \)-axis, from \( x = 0 \) to \( \infty \).
7. Determine the convergence or divergence of each of the following series. (Justify your answer, preferable by stating which tests for convergence or divergence you are using.)

(a) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}} \]

(b) \[ \sum_{n=0}^{\infty} \frac{1}{3^n - 1} \]

(c) \[ \sum_{n=1}^{\infty} (-1)^{n+1} \sin \left( \frac{1}{n} \right) \]

8. Find the degree 3 Taylor polynomial \( P_3(x) \) for \( f(x) = \frac{1}{x} \), centered at \( c = 2 \).
9. Consider the function \( f(x) = 17x^4 - 5x^3 + 3x^2 - x + 1 \). Make a guess as to the degree 4 MacLaurin polynomial of \( f(x) \). [What polynomial of degree 4 east approximates \( f(x) \), for values of \( x \) that are “close to” 0?]

10. Find the interval of convergence of each of the following power series.

(a) \[ \sum_{n=0}^{\infty} \frac{1}{(n + 1)!} x^{2n} \]

(b) \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \]

(c) \[ \sum_{n=0}^{\infty} \frac{x^n}{2^n(n + 1)} \]
11. Find a power series expression for $e^{-x^2}$.

12. Use the result of problem 11, to find $\int e^{-x^2} \, dx$. (Of course, the answer should be expressed as a power series.)

13. Find an equation for the hyperbola with foci at $(0, \pm 5)$ and with vertices at $(0, \pm 4)$.
14. Consider the parametric curve $x = e^t \cos t$, $y = e^t \sin t$. Find an equation for the tangent line to this curve at the point $(x, y)$ given by $t = 0$.

15. Find the length of the curve in problem 14, from $t = 0$ to $t = \pi/2$. 