ANALYTIC GEOMETRY AND CALCULUS II
Alternate Final Exam
Spring 1994

The point value of each problem is indicated in the left margin. You must show all of your work for full credit. Points will be deducted for faulty reasoning, for sloppy notation, and for failure to simplify answers, even if your answer is correct. You may use a calculator, your class notes, and any reference material. Explicitly cite, in some manner, any published formulae you use.

(7) 1. Find \( \frac{d}{dx}(xe^{x^2}) \).

Answer

(7) 2. Find \( \frac{d}{dx}\ln\sqrt{1 + x^2} \).

Answer
(7) 3. Find $\int x^2 e^{2x} \, dx$.

Answer $-C$

(7) 4. Find constants $A$ and $B$ so that

$$\frac{x}{(x - 1)(3x + 1)} = \frac{A}{x - 1} + \frac{B}{3x + 1}.$$

Answer $A = \text{________}$, $B = \text{________}$
5. Find \( \int \frac{dx}{x\sqrt{4x^2 - 9}}. \)

Answer \(-C\)

6. Express \( \int \frac{dx}{\sqrt{5 - 4x - 2x^2}} \) in the form \( \frac{1}{b} \int \frac{du}{\sqrt{a^2 - u^2}} \) where \( a \) and \( b \) are positive constants.

Answer \( \frac{1}{b} \int \frac{du}{\sqrt{25 - 9x^2}} \)
(7) 7. Evaluate \( \int_0^\infty \frac{x^2}{x^3 + 1} \, dx \), or determine that it does not exist.

Answer

(7) 8. What is the domain of \( f(x) = \sum_{k=0}^{\infty} k!(x + 2)^k \)?

Answer
(7) 9. Decide whether \( \sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n} \) converges absolutely, converges conditionally, or diverges.

Answer

(7) 10. Decide whether \( \sum_{n=1}^{\infty} (-1)^n \frac{n + 2}{n(n + 1)} \) converges absolutely, converges conditionally, or diverges.

Answer
11. Find a Maclaurin series for \( \int e^{-x^2} \, dx \).

Answer: \( -C \)

12. How many terms of the Maclaurin series for \( \int_{0}^{1/2} e^{-x^2} \, dx \) must be summed to approximate \( \int_{0}^{1/2} e^{-x^2} \, dx \) to within six decimal places — that is, we need |Error| < \( \frac{1}{2} \times 10^{-6} \)?

Answer: \( n \geq \)
(8) 13. Sketch the polar graphs of \( r = 2 \cos \theta \) and \( r = 1 \), and find the points of intersection.

Answer Intersection Points = ________________________________
14. Find the length of the cardoid \( r = \frac{1}{2} - \frac{1}{2} \cos \theta \left( = \sin^2 \left( \frac{\theta}{2} \right) \right) \).

Answer \( L = \)