## Introduction

What is Spanning Tree Modulus?
- Spanning tree modulus measures the “richness” of a family of spanning trees on a network
- $\rho \in \mathbb{R}_{>0}$ a set of edge weights
- $\mathcal{N}$ is the usage matrix for each edge in each spanning tree
- $\text{Mod}(T) := \min_{\rho \geq 1} \rho^T \rho$

Probabilistic Interpretation [1]
- $\mu \in \mathcal{P}(T)$ is a pmf on spanning trees
- $T$ is a random variable representing a spanning tree such that $\mu(T)$ is the probability of choosing $T$
- $\eta = \mathcal{N}^T \mu$ is the probability that an edge is in a random spanning tree

**minimize** $\sum_{e \in E} \mathbb{P}(e \in T)^2$

**subject to** $\mu \in \mathcal{P}(T)$
- $\mu^*$ is an optimal pmf on $T$
- $\eta^*$ is the optimal usage of edges in spanning trees

Homogeneous
- $\rho = (|V| - 1)^{-1}$ is always admissible
- A graph is homogeneous if $\rho^* = (|V| - 1)^{-1}$
- If a graph is homogeneous, $\text{Mod}(T) = \frac{|E|}{(|V| - 1)^2} \Rightarrow \eta^* = \frac{|V| - 1}{|E|}$

Regular, Connected
- A graph is $d$-regular if every node has degree $d$
- A graph is $k$-connected if a graph cannot be disconnected by removing fewer than $k$ nodes

## Results

**Theorem:** Each edge in a graph $G$ is in the same number of spanning trees if and only if $G$ is a uniform homogeneous graph.

**Theorem:** For $k \geq 3$, a $k$-regular, $k$-connected graph is homogeneous.

It is also proven that as $d$ gets large, $d$-regular graphs are almost surely $d$-connected [2]. Our theorem then proves that as $d$ gets large, $d$-regular graphs are almost surely homogeneous.

## References


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### Figures

**Figure 1:** 1-connected, 3-regular graph, labeled with $\eta$ values

**Figure 2:** Example of Deflation

- $\eta_{\text{max}} = 0.167$
- $\eta_{\text{max}} = 0.167$
- $\eta_{\text{max}} = 0.222$
- $\eta_{\text{max}} = 0.286$
- $\eta_{\text{max}} = 0.500$

**Figure 3:** Every cycle is uniform homogeneous.

**Figure 4:** The complete bipartite graph $K_{n,n}$ is homogeneous.