We derive the general state sum construction of two-dimensional topological quantum field theories (2-D TQFTs) with source defects on oriented curves, extending the state-sum construction from special symmetric Frobenius algebra for 2-D TQFTs without defects (cf. Lauda & Pfeiffer [2007]). From the extended Pachner moves (Crane & Yetter [2014]), we derive equations that we subsequently translate into string diagrams so that we can easily observe their properties. As in Dougherty, Park and Yetter [2014], we require that triangulations be flag-like, meaning that each simplex of the triangulations need to be flag-like.

We call $B$ a right Frobenius module if it is a module and comodule on Frobenius algebra $A$ satisfying: $(m \otimes 1_A)(1_B \otimes \delta) = (d_A)(m_A) = (1_B \otimes m_A)(\Delta \otimes 1_B)$.

We introduce a Frobenius-like on $C$: $(d_A)(m) = (1_C \otimes d_A)(m \otimes 1_A)$.

We introduce a special Frobenius-like on $C$: $(d_A)(m_A) = (1_B \otimes coeval)(1_B \otimes \Delta)(m_A \otimes 1_A)$.

Extended Pachner Move (2-4):

$\tau(m \otimes 1_B)(1_C \otimes tw_B)(1_C \otimes d)(1_B) = (m \otimes 1_B)(1_C \otimes m)(1_B)(1_C \otimes 1_B)(1_C \otimes d)(1_B)$

Main Theorem

If $a, b, c, \bar{a}, \bar{b}, \bar{c}$ are the families of structure coefficients for a special symmetric Frobenius algebra $A$, a right module and comodule $B$ over $A$, and a left (co)action of $C$ on $B$ (co)commuting with the (co)action of $A$, and these spaces satisfy the string equations shown, then

$\tau^{-|T_2|}\zeta^{-|T_2|}\sum_{\lambda \in \mathcal{T}_2} \frac{1}{\lambda^{e(\lambda(\sigma))}} \prod_{\sigma \in \mathcal{T}_2} a(\lambda(\sigma)) \ldots e(\lambda(\sigma))$

is independent of triangulation, where $\lambda$ are all suitable colorings of $T$, $a(\lambda(\sigma)) \ldots e(\lambda(\sigma))$ represents coefficients from the families $a \ldots e$ on $\sigma$ with coloring $\lambda$, and $T_2^0$ represents number of vertices off the curve and $T_2^0$ represents number of vertices on the curve.