The central theme running through the 2003 Prairie Lectures is the class of nonlinear PDEs whose prototype is the \( p \)-harmonic equation

\[
\text{div}\ |\nabla u|^{p-2} \nabla u = \text{div}\ f, \quad 1 < p < \infty
\]

A wider and more unifying framework will be established by means of differential forms and elliptic complexes, within which an associated \( q \)-harmonic dual system will exist, \( p+q = pq \). The selection of topics reflects years of our interaction with scholars and scientists in several disciplines of mathematics: geometric function theory, calculus of variations, nonlinear elasticity, composites with micro-structures, and so forth. Thus, \( p \)-harmonic analysis will tell us something about the existence of elastic deformations, their regularity and topological behavior. While the Sobolev space \( W^{1,p}(\Omega) \) is considered the natural domain of the \( p \)-harmonic operator, we shall depart from this space and move into the realm of so-called very weak solutions where important new applications lie. Viscosity solutions of the \( \infty \)-Laplacian will emerge in the lecture of Juan Manfredi. Participants may wish to consult with John Lewis about the parabolic case as well. Our study of the \( p \)-harmonic equations beyond their natural domain of definition brings us closer to new methods of PDEs such as nonlinear commutators of singular integrals, Hodge decomposition and much more. One might possibly say that the \( p \)-harmonic analysis is an excellent source of ideas upon which we build a viable theory of "nonlinear singular integrals", like Riesz transforms for the Laplacian. We shall face the truly fascinating task of capturing the fundamental role of the \( p \)-harmonic operators in nonlinear problems, a virtue Riesz transforms do not possess. However, we shall not leave behind the linear theory of singular integrals. Every effort will be made to reduce to a minimum the technical aspects in the interest of mathematical insights.

Welcome to the III Prairie Analysis Seminar