WEAK SOLUTIONS OF SEMILINEAR WAVE EQUATIONS

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Over the past decade there has been significant progress in the study of existence of weak solutions to the semilinear wave equation with power-like damping and source terms. The results mainly look at source and damping terms of polynomial growth and they prove either existence or blow up in finite time of the solution, depending on range of exponents and on the sign of the nonlinearities.

We will present a local in time existence theorem that brings improvements to recent work by J. Serrin, G. Todorova and E. Vitillaro, by extending the range of exponents and allowing more general nonlinearities. The equation under study has the form:

$$\begin{cases} u_{tt} - \Delta u + f(x, t, u) + g(x, t, u_t) = 0 \text{ a.e. in } \mathbb{R}^n \times [0, \infty); \\
 u|_{t=0} = u_0; \\
 u_t|_{t=0} = u_1, \end{cases}$$

(NLW)

where the functions $f$ and $g$ are polynomially bounded and $g$ is increasing in the last argument. The potential well technique due to L. E. Payne and D. H. Sattinger and a monotonicity argument due to J. L. Lions and W. Strauss are the main tools used.