Paper #1: What it means to understand mathematics

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What it means to understand mathematics

Skemp’s research analyzed the big picture of understanding mathematics. He described how crisis drives schema change. Most students want to learn only what they need to know in order to solve the problem. They do not want to learn too many concepts. Van Hiele, a mathematics teacher, has learned through observations of his students, that students have different levels of thinking. He realized that individual students each have a different level of understanding about different aspects of problem solving. Dubinsky had more specific objectives in describing the basic theoretical framework of the Action-Process-Object-Schema Theory (APOS). He believed that the educators should try to provide instructions of foundation of basic mathematical ideas on which the learners can build for future needs. He proposed that a student’s understanding of a mathematical concept could be applied to problem solving in different situations.

From a constructivist point of view, to understand mathematics is to assimilate it into an appropriate schema, or conceptual structure. From my experience, to understand mathematics is to know and be able to apply the concepts or to integrate new knowledge with existing knowledge about mathematics. If I can recognize and manipulate the concept that I have learned to solve the problem, then I consider that as understanding mathematics. However, I want to create my own theory after I have read about the research of others in the field. My theory is the basic theoretical framework of the Assess-Process-Learned-Apply (APLA). The APLA is the new theory, which would help educators determine what students understand and what they have learned. It also includes processes in academic development and assessment. The APLA theory is based on a student’s point of view, and it is the theory I want to apply in the future as a mathematics educator.
How this understanding develops

Assess students’ prior knowledge

In order to provide appropriate instruction to help students understand mathematics, educators need to know students’ knowledge levels. Students have to take an assessment test in order to be placed in an appropriate classroom. Prior knowledge that must be assessed includes language, culture, and subject area background. Students who have different cultures and backgrounds would have different ways to solve problems and understand the concepts.

Building on prior knowledge and experience are effective learning strategies because these allow students to learn mathematics by relating new knowledge to their previous experiences.

Process

Methods

To improve students’ cognitive skill, teachers should provide a strong foundation to move students toward higher-level thinking as a regular part of learning practice. Educators should allow students to use their general concept of mathematical knowledge to incorporate new concepts. Educators need to teach the basics of mathematics because students need to have a strong foundation in order to develop higher mathematical skills. Then, educators will teach students how to use their mathematical knowledge in real life. Students cannot just memorize the concepts in problem solving or formulas in algebra. They have to be able to recognize and understand the function of each concept, and they have to thoroughly assimilate the knowledge into their long-term understanding.

Strategies

- Hands-on activities create meaningful content because students learn best when they have an opportunity to use their five senses (taste, smell, touch, sight, and hearing). These
activities will be time consuming, but they will increase the students’ interest in learning mathematics.

- Cooperative skills allow students to learn from each other. Students like to work in groups when they get tired of listening to teachers lecture. This is an effective learning strategy to help students improve their social skills and learn how to interact and appreciate others. Teachers will have the chance to observe how students develop their decision-making in solving mathematics.

- **Shelter Instructions**

  For example, in geometry students have to draw pictures or diagrams to demonstrate the shape of a triangle, square, etc. In order to teach math in depth, teachers should provide many examples in which the same concept is demonstrated in a variety of ways. Students need sufficient practice so they can improve their thinking and recognizing skills.

  In algebra, students need to recognize the meaningful patterns of information in order to use the correct formulas to solve problems. Worksheets will help students practice and develop mathematic application skills. However, students need more examples to understand how the concept works. They need to know more than one way to solve problems. Teachers have to make the content meaningful by using examples which relate to the student’s prior knowledge or real life experience. For instance, to learn about money, teachers should use examples like, “if students had $5 and they spent $1 for pop, how much money did they have left?”

*Active Learning*

“Learning is important because no one is born with the ability to function competently as an adult in society” (*How People Learn*, chapter 3). Students need to put forth effort in developing their mathematics skills by learning the basic math such as algebra, geometry,
trigonometry, calculus, etc. In order to construct an active learning lesson, teachers need to know the students’ strengths and weaknesses. To make sure students understand the concept, instead of just memorizing the formula, teachers should provide at least one challenging problem to encourage students to manipulate all formulas and their knowledge to solve the problem. In the intermediate, elementary, middle school, and high school, students often try to memorize formulas. In college, students have many opportunities to practice critical problems, which requires more time and effort.

Collaboration is one of the active learning strategies. Students learn better when working with their classmates because they will have a chance to interact and share their knowledge with each other. This strategy also improves the students’ confidence and communication skills. Students will learn different techniques in problems solving. Collaboration is not only beneficial for students, but also convenient for teachers. Teachers have a chance to observe and motivate students while they working in groups. Teachers should allow students to construct their own mathematical strategies which they find out for themselves from working with others.

Mathematicians and mathematic educators have different expectations and styles of delivering mathematical knowledge to their students. For example, some teachers make the assumption that everyone in a Differential Equations class knows how to integrate by parts. Mathematic educators assess the students’ knowledge levels before they place the students in the appropriate classroom. Mathematic educators have more patience and effective instructional techniques in delivering their lesson plan. Understanding mathematics is a complicated process because teachers have to know what they need to teach and how to teach it, while students need to understand how to manipulate the concepts in order to develop their higher-level mathematical thinking skills.
Applying

DiSessa discussed stroke 3, which was about the culturally and experientially developed capability and interest to engage in extended, personally meaningful activities. Students who have different cultural backgrounds will have different ways in solving mathematic problems. The strategy that they use to apply or interpret the problem will be different. Teachers should allow them to work in their own way and then teachers can provide instructions after students work individually or in groups. For example, in the Foundation of Geometry class, my group had to meet outside of class to prove geometric theories. We work hard in order to find the correct proof. Everyone had to work together and share their inputs on how to solve the problems. Then we present our findings to the class after which my teacher provides a rationale for the proof. In this way, we learned from our mistakes. We apply what we learn and demonstrate the concepts to show how mathematics works. This is a great strategy for students to develop a higher knowledge levels and improve their cognitive development.
At least one specific example of how your theory work (apply it to learning decimals or functions for example)

I will be a secondary educator, and I would like to provide a lesson plan of how the English Language Learners (ELL) students learn problem-solving.

First, I will provide a worksheet with ten problems to solve I want to assess their prior knowledge. Secondly, I will provide instruction with three similar examples that demonstrate the concept. I also will create a new vocabulary list that will help students improve their mathematical language. I will highlight key words and provide examples not in the book. Students will work in groups to solve five problems, and they will share the answers with their classmates. In order to motivate students to participate in class activities, I will encourage students to ask questions if they have doubts. Problem-solving requires time, but students need to understand the concept more than just memorizing the formula. Problem-solving also develops students’ higher level thinking in mathematics. They will more fully understand concepts and improve their language acquisition. At times, students may already understand the concept, but they lack the appropriate terminology in English. Therefore, the ELL students need to understand the language before they can discuss the concept. The transition is not possible without the learning of a new language. In order to understand the problem-solving strategies, students need to transfer their prior knowledge in their native language to English. I have to progress more slowly and carefully check when reconstruction is needed. I want to make sure students understand mathematics as well as develop their English. I will start instruction by using visual aids, sheltered instructions, and hands-on activities to make the content easy to understand for the ELL students. After students pass the learning session, they will create one or two problems to solve. This activity allows students to use their knowledge by developing their
own questions and answers so that they can see how to solve problems without the book. I will collect best problems and create a formal assessment test. To make sure students learn the concepts, I will review the key content material/language that students need to know. I also will allow students to ask questions or work in groups to do their homework in class.

The APLA theory can apply to all subject areas of complex design. It is a mix of fundamental and practical strategies. The most important factor is that educators have to know about their students before they provide instruction. If they want students to be interested in learning mathematics, then they need to assess or observe what students like to study. Designing mathematic instruction for ELL students is not easy because I have to create effective teaching strategies in math and develop students’ language proficiencies as well. The concepts and the language both are needed to develop the higher level of thinking. Mathematics and language may draw on everyday experiences. The APLA theory is appropriate to use in mathematics, as this theory helps students understand how to apply and comprehended mathematics in their lives.
How your theory can be used (in instruction, assessment or research)

My theory is the basic theoretical framework of APLA (Assess-Process-Learned-Apply). The APLA can be used for all educational levels. In their instructions, educators will have a chance to get to know their students’ strengths and weaknesses are by providing an assessment test. The assessment test cannot determine the students’ intelligence about that subject matter because some students are not comfortable with taking tests. They perform poorly on tests, but they do very well in class. For this reason, the process is important so that everyone is on the same page. Students will then have a chance to create their own problem solving strategies based on what they have learned. Then they will demonstrate how to apply what they have learned from lecture. This theory provides a process of creating effective instruction.

To understand mathematics, students and educators have to assess what they know, what they want to learn, and how they will apply it in real life situations. Mathematics is not about computation (a calculation or computer can calculate) but, instead is a way of thinking about and understanding the world. It is a long process in developing mathematic skills. Everyone uses mathematics in their daily lives, but they do not realize it is mathematics. Mathematics can be performed formally and informally in academic performance, and the APLA takes this into consideration. Teachers will get a sense of what students know and what they can do as well as know their interests and passions. Effective teachers also help students build skills through self-assessment. Students will learn how to assess their own work after they have completed the APLA framework. Appropriately designed assessments can help teachers realize the need to rethink their teaching practices. Many mathematics teachers have been surprised at their students’ inabilities to answer a seemingly obvious question that assess their students’ understanding. These results motivate them to revise their instructional practices.
Opportunities for practicing teachers

Practicing teachers continue to learn about teaching in many ways. I want to learn from my own practice. I will gain new knowledge and understanding of my students and instructional methods by living the practical experiments that occur as a part of professional practice. The APLA is the framework that I want to follow when I become a teacher. It is a guide to provide step-by-step processing in my first year of teaching. I also want to learn from my own practice through different types of research and interactions with other people who have experience in teaching. The APLA will provide a good strategy for my teaching career in the future.

Assessment is the starting point for the educator to guide instruction in ways that target the cognitive development of the students regarding mathematics. In order to help my students understand mathematics, I will use the process of the APLA theory to make appropriate instructional decisions in order to assist students to construct their mathematical knowledge. To enhance learning, mathematic lessons frequently make use of math centers in which the students do a variety of activities. For example, using collaborative groups to solve problems will allow an opportunity for students to learn from each other. This will give me a chance to understand what students think about the mathematical ideas. I will select a solution that is actually incorrect to be presented. Then I will initiate a discussion of a common misconception about that problem. Students will learn more from their mistakes as it is said, “failure is the mother of success.” In application, students have an opportunity to construct their own learning style and apply the mathematics content to real world problem-solving.