**Synthesis Paper**

**What it Means to Understand a Physical Concept**

There are many theories that attempt to describe what it means to understand a mathematics or physical concept. Many of the theories we have read about and discussed throughout the course provide a theory of what it means to understand a mathematical concept. Those theories have advantages as well as disadvantages. Some of the ideas in my theory were taken from the theories we have read about (Skemp, Dubinsky, Bloom). In this synthesis paper, I will present a theory, which may not be the *only* theory that explains in detail what it means to describe a physical concept.

Throughout this synthesis paper, the terms ‘student’ and ‘subject’ are used interchangeably. Other terms that I will be using throughout the paper to describe what is means to understand a physical concept are included in the remainder of this paragraph. *Reflective abstraction* is the construction of mental object(s) and mental actions on the object(s). The object(s) may be external (visual) or internal (within the subject's mind). A *schema* is a conceptual structure of a subject's internal collection of object(s) and/or process(es). The conceptual structure may be simple as well as complex. While accounting for simple and/or complex structures of a subject’s schema(s), my theory will result in a *genetic decomposition* of the concept. A genetic decomposition may be beneficial to the subject and instructor during the course of instruction. The subject will know what is expected from the beginning of instruction, what they will learn, and how the topic depends upon previous learned material (possibly from
other courses as well). A genetic decomposition will also provide the instructor with specific goals for the unit plan or possibly the course (see Usefulness of the Theory in Instruction). According to Dubinsky (1991, p. 96), the genetic decomposition of a concept is a description of the mathematics involved and how a subject might make the construction(s) that would lead to an understanding of it. For my purpose, I will replace the term ‘mathematics’ with ‘physics,’ except when in a quotation.

For a subject to understand a physical concept, the instructor must have extensive content knowledge and organize instruction around the core concepts. To understand a physical concept, the subject must be able to recognize patterns, organize knowledge (chunking), recall previous knowledge rapidly, and strong metacognitive skills (Bransford, 2000, Chapter 3). Also, to understand a concept means to possess the ability to successfully solve problems within several contexts. The ability to solve problems in low and high levels of abstraction as well as concrete contexts is also crucial in understanding a physical concept. Once a physical concept is understood, the subject actively monitors his or her learning. The subject will also have the ability to compare and contrast various problems in one or more contexts. Another important aspect of understanding a physical concept is the subject’s motivation internally as well as motivation provided through instruction. Schoenfeld implies student’s need: resources, heuristics, control, and belief. To understand, the subject must have many resources that consist of knowledge learned from previous experiences. Heuristics is an ability to apply the resources to logically solve the problem. The control aspect according to Schoenfeld is the ability to evaluate one’s own thought process. If things are not working, the subject must realize they are not making progress and therefore proceed from a different resource or process. If the subject understands the concept, they shouldn’t choose paths to solve problems that are more time
Beliefs allow the subject to use all of the useful information and dispose any information that will not lead to a solution.

Reflective abstraction begins at an early age and continues throughout life. People in general, for my purpose students studying physics, utilize reflective abstraction no matter what their current understanding of the concept happens to be.

"Reflective abstraction is a concept introduced by Piaget to describe the construction of logico-mathematical structures by an individual during the course of cognitive development. Two important observations that Piaget made are first that reflective abstraction has no absolute beginning but is present at the very earliest ages in the coordination of sensori-motor structures (Beth & Piaget, 1966, pp. 203-208) and second, that it continues on up through higher mathematics to the extent that the entire history of the development of mathematics from antiquity to the present day may be considered as an example of the process of reflective abstraction (Piaget, 1985, pp. 149-150)." (Dubinsky, 1991, p.95).

A benefit of reflective abstraction is that people are continuously constructing physical structures of experiences in their everyday life. Therefore, reflective abstraction plays a crucial role in the understanding a physical concept. Student’s come to class everyday with previous conceptions that may be correct and/or incorrect from their everyday experiences. In order to minimize any misconceptions, the students must endure reflective abstraction throughout pre-instruction, instruction, and post-instruction.

Another benefit of reflective abstraction is that the student continues to utilize reflective abstraction in situations that increase in complexity as well as difficulty. According to Sousa (1995, p. 122), complexity establishes the level of thought while difficulty determines the amount of work within a level. The levels Sousa is referring to when defining complexity and difficulty are better known as Bloom's Taxonomy. The first level is referred to as knowledge and must exist before any understanding of a concept may occur. At the knowledge level, the subject recalls previously learned material. The second level of the hierarchy is comprehension. Comprehension, the lowest level of understanding according to Sousa, is when the subject
“makes sense” of the material by converting the content from one to another or by predicting consequences or effects. The third level in the hierarchy is the *application* level. Here the subject refers to the ability to use learned material in new situations with no direction at all or a minimum amount of direction from someone at a higher level. The fourth level is *analysis*, where the subject breaks material into its component parts so that its structure may be understood. The fifth level is *synthesis*. At the synthesis level, the subject organizes pieces of the lower levels (knowledge, comprehension, application, and analysis) together to form a plan that is new to the learner. Throughout the synthesis level, the subject uses creativity and emphasizes forming new structures. *Evaluation* occurs at the sixth (final) level. The ability to judge the value of the material based on specific criteria occurs during the evaluation level. Here the learner examines criteria from several schemas and selects the schemas that are most relevant to construct and gain knowledge. (Sousa, 1995, pp. 116-118). In order for a student to completely understand a physical concept, they must be at Bloom's sixth level of the Taxonomy. In order for the student to reach the sixth level, they must use reflective abstraction. Bloom's Taxonomy will not be the focus of this synthesis paper, but Dubinsky's theory of reflective abstraction and Skemp's theory of schema will be the focus. Bloom's Taxonomy does pertain to understanding a physical concept, that is the reason for including it in this paper, but the focus here is how the understanding develops within the levels.

Schemas are used to describe how the subject consistently responds to unique problems between levels of Bloom’s Taxonomy. Reflective abstraction is what the students use to advance to higher levels. I concur with Dubinsky when he writes:

“When the subject is successful, we say that the problem has been assimilated by the schema. When the subject is not successful then, in favorable conditions, her or his existing schemas may be accommodated to handle the new phenomenon. This is the constructive aspect of reflective abstraction...” (Dubinsky, 1991, p. 103).
Research (Skemp, 1987, p. 26) has shown schematically learnt material was not only better learnt, but better retained. “A subject's tendency to invoke a schema in order to understand, deal with, organize, or make sense out of a perceived problem situation is her or his knowledge of an individual concept in mathematics” (Dubinsky, 1991, p. 102). “Much too often mathematical formulas are applied like recipes in a complex reality that lacks any intermediate model to justify their use” (Freudenthal, 1991, p. 34). Simply applying a mathematical formula is not to be taken as understanding a physical concept.

**How the Understanding of a Physical Concept Develops**

The topic of discussion in this section will focus on how physical understanding develops. The understanding of a physical concept may occur at a very early age. To understand a physical concept, whether through an active/dynamic process (i.e. teaching-learning) or through a passive process (i.e. textbook), one must begin with a certain degree of mastery of the content. From Piaget's psychological viewpoint, new mathematical constructions proceed by *reflective abstraction* (Dubinsky, 1991, p. 98; Beth & Piaget, 1966, p. 205). I will expand Piaget’s statement above by stating the construction of a physical concept also proceeds by reflective abstraction. To develop new understanding of physical concepts, the subject proceeds through three kinds of abstraction.

According to Piaget, the kinds of abstraction are *empirical*, *pseudo-empirical*, and *reflective abstraction*. Empirical abstraction is the knowledge gained from an experience with an external object in which the subject performs (or imagines) actions on the object. Pseudo-empirical abstraction is the intermediate step between empirical and reflective abstraction that occurs after the action(s) have taken place on the object. During pseudo-empirical abstraction,
the subject engages with the external object and teases out properties of the actions introduced into the object during empirical abstraction. In the final step of developing new physical understanding, the subject undergoes reflective abstraction. During reflective abstraction, the subject reflects all actions on the object and develops a final schema (conceptual structure) of the knowledge gained. I wish to point out that reflective abstraction and the development of the schema occur internally and begin to occur at a very early age. Therefore, to change one's knowledge and build new understanding of a physical concept, the subject must proceed through at least one complete cycle of empirical, pseudo-empirical, and reflective abstraction and develop a schema throughout the abstraction process.

In order to begin the process of developing new understanding by reflective abstraction, assimilation of various schemas are needed for empirical abstraction (perform new actions on the external object). “According to Piaget, the first part of reflective abstraction consists of drawing properties from mental or physical actions at a particular level of thought” (Beth & Piaget, 1966, pp. 188-189). There is “a process which will become increasingly evident over time: the construction of new combinations by a conjunction of abstractions” (Piaget, 1972, p. 23). (Dubinsky, 1991, pp. 98-99). During the reflective abstraction process, the subject will internally build a schema or several schemas. Through time, the subject will discover, or be presented with, advanced topics in which the precious schemas may need to be modified. The necessary step to adapt the schema is included in the Skemp’s comment:

This brings us to a consideration of adaptability… What is then necessary is a change of structure in the schemas: they themselves must adapt. Instead of a stable, growing schema by means of which the individual organizes past experiences and assimilates new data, reconstruction is required before the new situation can be understood (Skemp, 1987, p. 27).

Dubinsky lists various kinds of construction in reflective abstraction, which is heavily based on the work by Piaget, for when the subject encounters a new topic (or physical content). When I
speak of an advanced topic, I am referring to one in which the subject has no existing schema based on any empirical or pseudo-empirical abstraction on the object (or concept).

The first kind of construction within reflective abstraction is *interiorization* and is referred to as “translating a succession of material actions into a system of interiorized operations” (Beth & Piaget, 1966, p. 206). The subject constructs an internal process (interiorization) as a way of making sense of the topic encountered when using symbols, communicating by language, and drawing diagrams when posed with an advanced topic. The subject may use a *coordination* of two or more processes to construct a new one. Also, the subject may *encapsulate* (convert) a dynamic process into an (static) object. The subject may learn to apply an existing schema to a wider collection of phenomena, which Dubinsky refers to as *generalizing* the schema. The final construction process of reflective abstraction according to Dubinsky (not Piaget) is *reversing*. The fifth (final) process is internal in which the subject reverses the original process to construct a new process. (Dubinsky, 1991, pp. 101-102). I will omit (agree with Piaget) the fifth construction process. In my view, the reverse of an original process is a part of ‘understanding’ the original process and therefore I do not consider reversing as a separate construction process.

Time is an important issue in understanding a physical concept. According to Sousa (1995, p. 122), the time to learn a concept is usually fixed even though students learn at different rates. The crucial aspect pertaining to the time issue of instruction and assessment is that the tie to learn a concept is usually fixed, but the amount of time for sorting (building schemas) varies (Sousa, 1995, p. 122). Providing social opportunities, relating content to everyday life, and actively monitoring one’s own learning also aids in understanding a physical concept (Bransford, 2000, Chapter 3).
The construction of various physical concepts may be described through each of the four kinds of reflective abstraction: interiorization, coordination, encapsulation, and generalization. The subject uses previous schemas or a combination of several previous schemas to proceed through empirical abstraction to reflective abstraction and, ultimately, understand a physical concept. The subject will now be able to utilize the genetic decomposition of the concept they have learned and relate the small details (all parts building down to the goal) to a larger picture (how all the parts of the decomposition tie in with one another).

**Example of the Theory**

I would like to make a note at this point that the above theory has not been tested. The content I chose for a specific example of how the theory works is based on the concept of resultant displacement in two-dimensions. The subject must begin with some mastery of the original content. The original content includes addition/subtraction/multiplication/division, trigonometry skills (graphical representation of a right triangle, Pythagorean theorem, and the cosine, sine, and tangent functions), coordinate systems, and vectors (definition of a vector, components of a vector, and vector arithmetic).

The subject then begins the process of understanding the physical concept of resultant displacement in two-dimensions by interacting with the object(s), in this specific example vectors are the objects. The actions the subject performs on the objects during empirical abstraction is placing the vectors on a single coordinate system or placing each vector on its own coordinate system. In order to achieve the above, the subject must have a schema developed for performing the task of placing vectors on a coordinate system, which is part of previously learned content.
The next step in understanding the physical concept of resultant displacement in two-dimensions is pseudo-empirical abstraction. During pseudo-empirical abstraction, the subject engages with the external objects and teases out properties of the actions introduced into the objects during empirical abstraction. Here the subject realizes that each vector placed on its own coordinate system or all on a single coordinate system is necessary in order to add all of the vector components. The subject must have a schema of vector components prior to performing pseudo-empirical abstraction.

During reflective abstraction, which is the next step in understanding resultant displacement in two-dimensions, the subject must have schemas for addition/subtraction/multiplication/division and trigonometry (graphical representation of a right triangle, Pythagorean theorem, and the cosine, sine, and tangent functions). The subject then uses the construction process of coordination to combine the two schemas into a larger schema, vector arithmetic. Before the subject can add the component in the two directions, he or she must move back and restart the abstraction cycle to empirical abstraction in order to find the actual components of each individual vector. Again, the objects are considered the vectors. The actions placed on the objects are constructing right triangles of each individual vector. The subject may then use his or her schemas of trigonometry to calculate the components of each individual object. Once the components of each object are calculated, the subject is now ready to utilize reflective abstraction and add the components in each direction of all objects.

During the second reflective abstraction process, the subject makes sense (interiorization) of adding the components in the individual direction. The subject then uses coordination of two schemas based on the Pythagorean theorem and geometrical representation of a right triangle to calculate the final resultant displacement. Once again the subject refers to an existing schema.
that the resultant displacement is a vector quantity and makes sense of the process (interiorization).

Understanding the physical concept of resultant displacement in two-dimensions is now complete and the subject will be able to utilize genetic decomposition with or without help from the instructor (see Figure below). The subject can now apply the large, and new schema of the resultant displacement in two-dimensions and relate the small details (all parts building down to the goal) to a larger picture (how all the parts of the decomposition tie in with one another) to resultant displacement in three-dimensions (generalization).
Usefulness of the Theory in Instruction

The above format of the theory is beneficial to learning as well as aiding instruction. Teaching students general problem solving strategies (heuristics, see Pólya) is an important part of instruction. Having a general strategy enables subjects to maximize the use of reflective abstraction and generalize concepts to build schemas. Sousa comments (1995, p. 122) that students of various abilities learn a concept at a fixed time even though various students learn at different rates. Therefore, the assessment of instruction must not be as frequent in order to provide learners of slow and fast learning abilities an adequate amount of time to process the information. Teachers must give the students an adequate amount of time for practice and time to learn the process when solving problems (Bransford, 2000, Chapter 3). During this time of process, the instructors must make sure that schematic learning, not just rote memorization and manipulations of symbols, is taking place (Skemp, 1987, p. 34).

The instructor may begin by assessing his or her students to evaluate what misconception and/or conceptions the students have at the beginning and throughout each topic discussed. A way instructors may gain the current information of their students is to pose questions for the students to solve during lecture (i.e. Personal Response System). Using the PRS system provides immediate feedback allows the instructor to know their students ability at a specific instant throughout lecture. The instructor may then plan the appropriate times in which a majority of the students are ready to build new knowledge based on their current conceptions. Van Hiele (1986, p. 45) specifies an important aspect of instruction: “A teacher, … should address himself to the pupils in a language they understand.” If the students do not understand and comprehend the material being presented at a certain stage of learning a physical concept, whether during lecture or some other form of instruction, the instructor has failed to serve his or her purpose of being an
educator. Therefore it is a necessity that the instructor speaks the language the students are familiar with.

Another aspect the instructor must account for is that a number of subject’s have very different goals. Therefore, the instruction should try to lay well-structured foundation of the basic physical concepts in which the learner can build in whatever direction becomes necessary (Skemp, 1987, p. 34). Take advantage of the power of positive transfer by integrating the concepts learned with previously-taught material and connecting them to appropriate concepts in other curriculum areas (Sousa, 1995, p. 123). To accomplish all of the above, the teacher should be prepared, organized, and willing to look further ‘down the road’. A very simple way, I consider it simple because the final result of my theory leads to the same point, to help students construct understanding of physical concepts is in providing a genetic decomposition of the content being discussed. Providing a genetic decomposition allows the unmotivated subjects to see the end result after days or weeks of instruction and for the instructor to have clearly defined goals for his or her students.

As a final comment in the current section, I would like to stress that the instructor must be willing to postpone the content to be learned by the subjects if the subjects do not have an understanding of the previously learned material. Progressing through instruction when the students are not ready will hinder knowledge, motivation, and success of the students.

**Summary**

To understand a physical concept is not an easy task for the instructor as well as the subject. If the instructor is well prepared and the subject is motivated, understanding will come forth and provide success for the subject as well as the instructor. To understand a physical
concept begins with abstraction. “Empirical and pseudo-empirical abstraction draws knowledge from objects by performing (or imagining) actions on them. Reflective abstraction interiorizes and coordinates these actions to form new actions and, ultimately new objects (which may no longer be physical but rather mathematical such as a function or a group). Empirical abstraction then extracts data from these new objects through mental actions on them, and so on” (Dubinsky, 1991, p. 98). “A subject's tendency to invoke a schema in order to understand, deal with, organize, or make sense out of a perceived problem situation is her or his knowledge of an individual concept in mathematics” (Dubinsky, 1991, p. 102). The above occurs through the process of reflective abstraction, which includes the construction processes: interiorization, coordination, encapsulation, and generalization. A genetic decomposition may be beneficial to the subject and instructor prior to and during the course of instruction. The subject will know what is expected from the beginning of instruction and a genetic decomposition will also provide the instructor with specific goals for the unit plan or possibly the course.

References


