PROBLEM 1. In each of the following cases we are dealing with a system $Ax = b$. Each case has a different coefficient matrix $A$ and a different right hand side vector $b$. Also, in each case we are told what $\text{rref}([A \ b])$ is. Now, in each case determine the following:

1. Size of the matrix $A$ (number of rows and number of columns)

2. An expression for the solution(s) to the system (if the system has no solutions, point that out). In the case of having infinitely many solutions, also indicate the number of free variables.

Case 1.

```matlab
>> rref([A b])
ans=  
1 0 0 15 7
0 1 0 4 0
0 0 1 -10 -4
```

Case 2.

```matlab
>> rref([A b])
ans=  
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
0 0 0 0
```

Case 3.

```matlab
>> rref([A b])
ans=  
1 0 0 0 0
0 1 0 0 0
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
```

1
Case 4.

```matlab
>> rref([A b])
ans=
1 0 0 17
0 1 0 4
0 0 1 -11
```

Case 5.

```matlab
>> rref([A b])
ans=
1 2 0 0 -7 -32
0 0 1 0 8 25
0 0 0 1 -3 -10
```

**PROBLEM 2.** The matrix

\[
A = \begin{pmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}
\]

is the adjacency matrix of a graph \(G\). Is \(G\) a directed or undirected graph? Draw the graph \(G\). How many walks of length 3 joining the vertex 2 and the vertex 4 are there?

**PROBLEM 3.** For what values of \(k\) is the matrix

\[
B = \begin{pmatrix}
k^2 & 16 \\
k & k
\end{pmatrix}
\]

singular? If \(k = 1\), how many solutions to \(Bx = \begin{pmatrix} 1 \\ 2 \end{pmatrix}\) are there?

**PROBLEM 4.** In this problem \(A\), \(B\) and \(C\) are matrices. Suppose that the size of \(AB\) is \(4 \times 3\) and the size of \(BC\) is \(6 \times 5\). What’s the size of \(A\)?

**PROBLEM 5.** In No-Mid-Class City there are two neighborhoods: Fancy Woods and Shanty Hills. Each year, 40% of the residents of Fancy Woods move to Shanty Hills and 10% of the residents of Shanty Hills move to Fancy Woods. If this year there are exactly 1,000 residents in each neighborhood. What will be the distribution of residents in 2 years? Write down the step matrix.

**PROBLEM 6.** The following system

\[
\begin{align*}
\begin{cases}
  x_1 + x_4 &= 450 \\
  -x_1 + x_2 &= 250 \\
  x_2 - x_3 &= 300 \\
  x_3 + x_4 &= 400
\end{cases}
\end{align*}
\]
describes a traffic flow network. Draw the network. Indicate the matrix of coefficients \( A \) and the right hand side vector \( b \) for this case. If we know that

\[
\begin{array}{cccc}
1 & 0 & 0 & 1 & 450 \\
0 & 1 & 0 & 1 & 700 \\
0 & 0 & 1 & 1 & 400 \\
0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

What is the traffic situation when \( x_4 = 100 \)? (That is, determine the values of \( x_1 \), \( x_2 \), and \( x_3 \).)