MATH 551 - APPLIED MATRIX THEORY
Fall 2006
IN-CLASS TEST 1 (150 points)

NAME:

PROBLEM 1. (40 points) In each of the following cases we are dealing with a system $Ax = b$. Each case has a different coefficient matrix $A$ and a different right hand side vector $b$. Also, in each case we are told what $\text{rref}(A \ b)$ is. Now, in each case determine the following:

(a) Size of the matrix $A$ (number of rows and number of columns) (1 point for each case)

(b) An expression for the solution(s) to the system (if the system has no solutions, point that out). In the case of having infinitely many solutions, also indicate the number of free variables. (7 points for each case)

Case 1.

$>> \text{rref}([A \ b])$

ans=

\begin{align*}
1 & 0 & 0 & 1 & 8 \\
0 & 1 & 0 & 4 & 1 \\
0 & 0 & 1 & -10 & -1 \\
\end{align*}

Case 2.

$>> \text{rref}([A \ b])$

ans=

\begin{align*}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{align*}

Case 3.

$>> \text{rref}([A \ b])$

ans=

\begin{align*}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{align*}

1
Case 4.

```matlab
>> rref([A b])
ans=
    1 0 0 1
    0 1 0 4
    0 0 1 -4
```

Case 5.

```matlab
>> rref([A b])
ans=
    1 2 0 0 -1 -3
    0 0 1 0 8 5
    0 0 0 1 -3 0
```

**PROBLEM 2.** (**25 points**) The matrix

$$A = \begin{pmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

is the adjacency matrix of a graph $G$.

(a) Is $G$ a directed or undirected graph? (**5 points**)  
(b) Draw the graph $G$. (**5 points**)  
(c) How many walks of length 3 from vertex 1 to vertex 4 are there? (**15 points**)
PROBLEM 3. (15 points)
(a) For what values of \( k \) is the matrix
\[
B = \begin{pmatrix}
  k^2 & 2k \\
  8 & k \\
\end{pmatrix}
\]
singular? (10 points)
(b) If \( k = 1 \), how many solutions to \( Bx = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \) are there? (5 points)

PROBLEM 4. (15 points) In this problem \( A, B \) and \( C \) are matrices. Suppose that the size of \( AB \) is \( 4 \times 3 \) and the size of \( BC \) is \( 6 \times 5 \). What’s the size of \( A^T \)?

PROBLEM 5. (15 points) The following system
\[
(S) \begin{cases}
  x_1 + x_4 = 100 \\
  -x_1 + x_2 + x_4 = 50 \\
  x_2 - x_3 = 30
\end{cases}
\]
describes a traffic flow network.
(a) Indicate the matrix of coefficients \( A \) and the right hand side vector \( b \) for this case. (5 points)
If we know that
\[
>> \text{rref}([A \ b])
\]
\[
\begin{array}{cccc}
  1 & 0 & 0 & 1 & 100 \\
  0 & 1 & 0 & 1 & 150 \\
  0 & 0 & 1 & 1 & 120
\end{array}
\]
(b) What is the traffic situation when \( x_4 = 20 \)? (10 points)
PROBLEM 6. (40 points) A community classifies its land in one of three ways; urban (U), agricultural (A), and forested (F). A recent survey showed that 10% was classified U, 40% classified A, and 50% classified F. In a survey carried out five years later it was found that 80% of the urban land remained urban, 15% had become agricultural, and 5% had become forested. The remaining survey data are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>A</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>80%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>A</td>
<td>15%</td>
<td>90%</td>
<td>5%</td>
</tr>
<tr>
<td>F</td>
<td>5%</td>
<td>5%</td>
<td>90%</td>
</tr>
</tbody>
</table>

(a) Find the step matrix $A$ for the discrete dynamical system that models the classification of land use from survey to survey. (15 points)

(b) Assuming that trend remains the same, predict the classification of land use for the next three surveys. (25 points)