PROBLEM 1. Let \( T : \mathbb{R}^2 \to \mathbb{R}^3 \) be the linear transformation given by
\[
T(x_1, x_2) = \begin{pmatrix} 2x_1 + x_2 \\ 3x_1 \\ 4x_2 \end{pmatrix}
\]
(i) Find a \( 3 \times 2 \) matrix \( A \) such that \( Tx = Ax \) for every \( x = (x_1, x_2) \in \mathbb{R}^2 \).
(ii) Find a basis for \( \text{range}(T) \). Is \( T \) onto?
(iii) Let \( b = [0 \ 0 \ 0]' \). If we have that
\[
\text{rref}([A \ b])
\]
\[
\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}
\]
What is \( \ker(T) \)? Is \( T \) one-to-one?

PROBLEM 2. Find \( x \) and \( y \) such that the matrix \( Q \) given by
\[
Q = \begin{pmatrix} x & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{-\sqrt{3}}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} & \frac{-\sqrt{3}}{\sqrt{6}} & y
\end{pmatrix}
\]
is orthogonal.

PROBLEM 3. Find the projection of the vector \( v = [1, \ 0, \ -1, \ 4]' \) onto the subspace spanned by the vectors \( u_1 = [1, \ 1, \ 1, \ 1]' \) and \( u_2 = [1, \ -1, \ 1, \ -1]' \).

PROBLEM 4. Use cross products to find the area of the triangle on the plane whose vertices are \( P = (1, 1), \ Q = (3, 5) \) and \( R = (6, 4) \).

PROBLEM 5. Suppose that we have three square matrices \( A, B, \) and \( C \) such that \( A \) is invertible and \( AB = CA^T \). Here \( A^T \) is the transpose of \( A \). Find \( \det(C) \) assuming that \( \det(B) = 4 \). Is \( C \) invertible?

PROBLEM 6 Is \( T \) linear?

(i) YES - NO. \( T(x_1, x_2) = x_1 + 2x_2 + 3. \)
(ii) YES - NO. \( T(x_1, x_2, x_3) = [x_1 - x_2, 3x_1 + 2x_2]' \).

Is \( U \) subspace?

(i) YES - NO. \( U = \{ x = [x_1, x_2, x_3]' \in \mathbb{R}^3 : x_1 + x_3 = 0 \} \).

(ii) YES - NO. \( U = \{ x = [x_1, x_2]' \in \mathbb{R}^2 : x_2 = 2x_1 \} \).

**PROBLEM 7.** In the following cases explain why the matrix \( A \) in question is diagonalizable or not.

**Case 1.**

\[ P \text{span}\{eig(A)\} = \begin{bmatrix} 0 & 0 & 0.4841; 0 & 0.1961 & -0.8713; 1.0000 & -0.9806 & 0.0807 \end{bmatrix} \]

\[ D = \begin{bmatrix} 7 & 0 & 0; 0 & 6 & 0; 0 & 0 & 1 \end{bmatrix} \]

What is \( \text{det}(A) \)?

**Case 2.**

\[ P \text{span}\{eig(A)\} = \begin{bmatrix} 0 & -0.7071i & 0 + 0.7071i; 0.7071 & 0.7071 & 0; 0 & 0 & 1.0000 \end{bmatrix} \]

\[ D = \begin{bmatrix} 1.0000 + 5.0000i & 0 & 0; 0 & 1.0000 - 5.0000i & 0; 0 & 0 & 1.0000 \end{bmatrix} \]

**Case 3.**

\[ P \text{span}\{eig(A)\} = \begin{bmatrix} 0 & 0.7071; 0 & 0.7071; 0.7071 & 0.7071; 1.0000 & 0.7071 & 0 \end{bmatrix} \]

\[ D = \begin{bmatrix} 1 & 0 & 0; 0 & -1 & 0; 0 & 0 & 1 \end{bmatrix} \]

**Case 4.**

\[ P \text{span}\{eig(A)\} = \begin{bmatrix} -0.7071 & 0.7071; 0.7071 & 0.7071 \end{bmatrix} \]

\[ D = \begin{bmatrix} 0 & 0; 0 & 2 \end{bmatrix} \]

**Case 5.**

\[ P \text{span}\{eig(A)\} = \begin{bmatrix} 1.0000 & 1.0000; 0 & 0.0000 \end{bmatrix} \]

\[ D = \begin{bmatrix} 1 & 0; 0 & 1 \end{bmatrix} \]