You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded in red ink! There will be no mercy for the weak. Mathematics is cumulative. Deal with it. What you don’t know will hurt you. You need to be able to make simple and/or standard simplifications. In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (The only exception being that since this test is longer than the actual test, it makes sense for you to take it in more than one sitting.)

1. State the quadratic formula and state when it applies!

2. Graph the following functions. You should include sign diagrams for every function. Any zeros, vertical asymptotes, and/or axes of symmetry should be listed. Based on the graph you draw state whether your function satisfies the horizontal line test.

   (a) \[ y = 3x - 2 \]
   (b) \[ y = x^2 - 5x + 6 \]
   (c) \[ y = \frac{x-2}{x+3} \]
   (d) \[ y = \frac{2}{3}x + 4 \]
   (e) \[ y = x^2 + 3x - 11 \]
   (f) \[ y = \frac{(x+3)(5-x)}{(x^2+2)(x-1)} \]
   (g) \[ y = \frac{(x+1)^2(3-x)}{x^3(x+4)} \]
   (h) \[ y = \frac{2}{x+3} - \frac{3}{x-4} \]

3. Solve for \( x \) and \( y \):

   (a) \[ \begin{align*}
   2x + y &= 7 \\
   x - y &= -1
   \end{align*} \]
   (b) \[ \begin{align*}
   2x - 3y &= 1 \\
   3x + 2y &= 17
   \end{align*} \]
   (c) \[ \begin{align*}
   4x + 3y &= 2 \\
   2x + 9y &= -1
   \end{align*} \]
4. Find \( f(g(x)) \), \( g(f(x)) \), and \( f(x)g(x) \) if

(a) \( f(x) = 2x, \ g(x) = x - 5 \).
(b) \( f(x) = 3x + 2, \ g(x) = -2x + 5 \).
(c) \( f(x) = 2x + 3, \ g(x) = 2x^2 - 5x \).
(d) \( f(x) = \frac{2}{x-1}, \ g(x) = x^2 + 3 \).
(e) \( f(x) = x^3, \ g(x) = x^4 \).

5. Assume \( y = f(x) \). Find \( f^{-1}(y) \) if

(a) \( f(x) = 2x + 3 \).
(b) \( f(x) = \frac{3x-2}{7} \).
(c) \( f(x) = \frac{2}{5}(x - 32) \).
(d) \( f(x) = 8x^3 - 27 \).
(e) \( f(x) = \frac{3}{2x+5} \).

6. Word problems:

(a) The temperature, \( u \), of water in a pot on the stove \( t \) minutes after it is placed there is given by \( u = h(t) = 92 + 20t \) for \( 0 \leq t \leq 6 \). Find the time as a function of temperature.

(b) My distance, \( d \), from Manhattan on my trip to Austin, \( t \) hours after leaving is given by \( d = h(t) = 60t \) for \( 0 \leq t \leq 10 \). Find the time as a function of distance.

7. True or False:

(a) The graph of every function satisfies the vertical line test.
(b) The graph of every function satisfies the horizontal line test.
(c) If \( y = f(x) \) and \( x = f^{-1}(y) \), then \( x = f^{-1}(f(x)) \) for every \( x \) in the domain of \( f \).
(d) If \( y = f(x) \) and \( x = f^{-1}(y) \), then \( y = f(f^{-1}(y)) \) for every \( y \) in the domain of \( f^{-1} \).

(e) \[
    f^{-1}(y) = \frac{1}{f(y)} .
\]

(f) \[
    (f(y))^{-1} = \frac{1}{f(y)} .
\]

(g) If \( y = f(g(x)) \) and both \( f \) and \( g \) are invertible, then

\[
    x = f^{-1}(g^{-1}(y)) .
\]

8. Does the function

\[
    y = \frac{x + 3}{(x - 5)^2}
\]

have an inverse? Justify your answer!

9. The quadratic function

\[
    y = Q(x) = x^2 - 4x + 1
\]

does not have an inverse if we consider it on the domain of all real numbers. Find a smaller domain containing \( x = 0 \) where the restricted function is invertible and find the inverse on this domain. For what \( y \)-values does the inverse function make sense? What is the \( y \)-value of the vertex of the parabola? Redo all of this problem for the quadratic function

\[
    y = Q(x) = 2x^2 + 8x - 3 .
\]