1.

\[
\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7}{5 \cdot 7} + \frac{3 \cdot 5}{7 \cdot 5} = \frac{14}{35} + \frac{15}{35} = \frac{29}{35}.
\]

\[
\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}.
\]

\[
\frac{2}{5} = \frac{2 \cdot 7}{5 \cdot 3} = \frac{14}{15}.
\]

\[
\frac{7}{18} = \frac{7 \cdot 18}{18 \cdot 5} = \frac{7}{5}.
\]

\[
2 - 3 + 4 - 5 \times 2 + 6 \times 3 = 2 - 3 + 4 - 10 + 18 = 11
\]

Note: Addition does not come before subtraction in the order of operations. They happen from left to right. If you prefer, view the subtractions as addition of negative numbers, and then by the commutative property and the associative property you can add all of the positives first (in this case to get 24) add all of the negatives next (in this case to get -13), and simply add the results. 24 + (-13) = 11.

\[
2 \times 3 - 4 \times 5 + 6 \times 7 = 6 - 20 + 42 = 28.
\]

\[
234 \times 427 + 234 \times 573 = 234 \times (427 + 573) = 234 \times 1000 = 234,000.
\]
2. Express each fraction as a decimal:

\[
\frac{3}{8} = .375 \quad \frac{51}{34} = 1.5 \quad \frac{3}{7} = .428571 .
\]

3. Simplify and/or compute by long division:

\[
\frac{.01}{.0001} = 100 \quad \frac{.00001}{.001} = \frac{1}{100} \quad \text{or} \quad .01 .
\]

\[
\frac{34,128}{54} = 632 \quad \frac{129,584}{364} = 356 .
\]

\[
\frac{248}{7} = 35\frac{3}{7} .
\]

4. Simplify the following expressions:

\[
\frac{2^3 x^2 y^5}{2^2 x^5 y^3} = \frac{2^1 x^{-3} y^2}{x^3} = \frac{2y^2}{x^3} .
\]

\[
\frac{(x^2 y^4)^3}{(x^5 y^3)^2} = \frac{x^6 y^{12}}{x^{10} y^6} = \frac{y^6}{x^4} .
\]
\[
\frac{(2^5w^2x^3y^{-2}z^{-4})^2}{(2^3w^{-1}x^2y^{-3}z^{-4})^3} = \frac{2^{10}w^4x^6y^{-4}z^{-8}}{2^9w^{-3}x^6y^{-9}z^{12}} = 2^1w^7x^0y^5z^{-20} = \frac{2w^7y^5}{z^{20}}.
\]

\[
\frac{x^{-1} - y^{-1}}{x^{-1} + y^{-1}} = \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} = \frac{1}{x} \cdot \frac{y - 1}{y} \cdot \frac{x}{x} = \frac{y-x}{xy} \cdot xy = \frac{x+y}{x+y}.
\]

5. Express the following repeating decimals as fractions:

Let \( S := 2.7\overline{3} \).

Then \( 10S = 27.\overline{3} = 27.33 \).

So \( 9S = 10S - S = 27.33 - 2.73 = 24.6 \).

Thus \( S = \frac{24.6}{9} = \frac{246}{90} = \frac{123}{45} = \frac{41}{15} \).

Let \( T := 2.521\overline{6} \).

Then \( 10T := 25.21\overline{6} = 25.216\overline{6} \).

So \( 9T = 10T - T = 25.216\overline{6} - 2.521\overline{6} = 22.695 \).

Thus \( T = \frac{22.695}{9} = \frac{22695}{9000} = \frac{4539}{1800} = \frac{1513}{600} \).

Let \( U := 4.1\overline{35} \).

Then \( 100U = 413.5\overline{35} \).

So \( 99U = 100U - U = 413.5\overline{35} - 4.1\overline{35} = 409.4 \).
Thus \( U = \frac{409.4}{99} = \frac{4094}{990} = \frac{2047}{495} \).

6. Use a sketch and the fact that the area of a rectangle is its length times its width to justify:

   (a) The distributive property.

   (b) The “FOIL” rule.
7. Justify the following procedure: To square a two digit number ending in 5 (like 65), take the first digit and multiply it by one more than itself, then multiply by 100, and add 25 to the result.
In the case of $65^2$ here is the justification:

$$65^2 = (60 + 5)(60 + 5)$$
$$= 60 \cdot 60 + 60 \cdot 5 + 60 \cdot 5 + 5 \cdot 5$$
$$= 60 \cdot (60 + 5 + 5) + 25$$
$$= 60 \cdot 70 + 25$$
$$= 6 \cdot 10 \cdot 7 \cdot 10 + 25$$
$$= (6 \cdot 7) \cdot 100 + 25 .$$

More generally, numbers of this type can all be written as $10x + 5$ so here is how it looks in the general case:

$$(10x + 5)^2 = (10x + 5)(10x + 5)$$
$$= 10x \cdot 10x + 10x \cdot 5 + 10x \cdot 5 + 5 \cdot 5$$
$$= 10x \cdot (10x + 5 + 5) + 25$$
$$= 10x \cdot (10x + 10) + 25$$
$$= 10x \cdot (10x + 10 \cdot 1) + 25$$
$$= 10x \cdot (10(x + 1)) + 25$$
$$= x \cdot 10 \cdot (x + 1) \cdot 10 + 25$$
$$= x(x + 1)100 + 25 .$$

8. State which of the following polynomials can be factored, and factor those expressions.

(a) $x^2 - y^2 = (x - y)(x + y)$
(b) $x^2 + y^2$ cannot be factored.
(c) $x^2 + 2xy + y^2 = (x + y)^2$

9. By using part (a) of the last problem with $x = 70$ and $y = 3$ we have

$$67 \times 73 = (70 - 3)(70 + 3) = 70^2 - 3^2.$$
10. Solve the following:

(a) 42 is what percent of 28?

\[ 42 = \left( \frac{x}{100} \right) 28 \]

\[ \frac{42}{28} = \frac{x}{100} \]

\[ 3 = \frac{x}{100} \]

\[ 150 = x \]

(b) What percent of 144 is 96?

\[ \left( \frac{x}{100} \right) 144 = 96 \]

\[ \frac{x}{100} = \frac{96}{144} = \frac{2}{3} \]

\[ x = \frac{200}{3} = 66 \frac{2}{3} \]

(c) How many square inches make up a square foot?

\[ 1\text{ft}^2 = 1\text{ft}^2 \cdot \frac{12\text{in}}{1\text{ft}} \cdot \frac{12\text{in}}{1\text{ft}} = 1\text{ft}^2 \cdot \frac{144\text{in}^2}{1\text{ft}^2} = 144\text{in}^2 . \]

(d) Express 2.3 days using whole numbers of days, hours, minutes, and seconds.

Scrapwork:

\[ .3 \text{ days} = .3 \text{ days} \cdot \frac{24 \text{ hrs}}{1 \text{ day}} = 7.2 \text{ hrs} . \]

\[ .2 \text{ hrs} = .2 \text{ hrs} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 12 \text{ min} . \]

So, 2.3 days is 2 days, 7 hours, and 12 minutes.
(e) If I say that something is within 100% of 400, then how large can it be? How small can it be?
Since 100% of 400 is 400, the number can be as large as 800 and as small as 0.

(f) 45 miles per hour is how many miles per minute?

\[
\frac{45 \text{ mi}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{45 \text{ mi}}{60 \text{ min}} = \frac{3 \text{ mi}}{4 \text{ min}} = \frac{\frac{3}{4} \text{ mi}}{1 \text{ min}}
\]

(g) How long will it take me to go 10 miles if I am traveling at 75 miles per hour?

Two ways: First way and the most straightforward - since distance equals rate times time, we have

\[
10 \text{ mi} = \frac{75 \text{ mi}}{1 \text{ hr}} \cdot t.
\]

\[
t = \frac{10 \text{ hr}}{75} = \frac{2 \text{ hr}}{15} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{120 \text{ min}}{15} = 8 \text{ min}.
\]

Second way is to observe

\[
\frac{75 \text{ mi}}{1 \text{ hr}} = \frac{75 \text{ mi}}{60 \text{ min}} = \frac{5 \text{ mi}}{4 \text{ min}},
\]

so 5 miles in 4 minutes and therefore 10 miles in 8 minutes.

11. When John is told to divide \( \frac{3}{5} \) by \( \frac{4}{7} \) he computes it by doing the multiplication

\[
\frac{3}{5} \cdot \frac{7}{4}
\]

Justify this procedure.

\[
\frac{\frac{3}{5}}{\frac{4}{7}} = \frac{\frac{3}{5}}{\frac{4}{7}} \cdot \frac{\frac{7}{4}}{\frac{1}{1}} = \frac{\frac{3}{5} \cdot \frac{7}{4}}{1} = \frac{3}{5} \cdot \frac{7}{4}
\]
12. Simplify the following:

\[
\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}.
\]

\[
\sqrt{175} = \sqrt{25 \cdot 7} = 5\sqrt{7}.
\]

\[
\sqrt{98} = \sqrt{49 \cdot 2} = 7\sqrt{2}.
\]