1. Find and classify as stable, unstable, or unable to tell, the equilibria for the following systems:

   a. \[
   \frac{dx}{dt} = x^2 + y^2 - 2, \quad \frac{dy}{dt} = x - y.
   \]

   b. \[
   \frac{dx}{dt} = 100x - 5xy, \quad \frac{dy}{dt} = -10y + 2xy.
   \]

2. The predator-prey model is a non-linear autonomous model of two interacting species, whose populations at time \( t \) are given by \( x(t) \) and \( y(t) \). An example of the model is

   \[
   \frac{dx}{dt} = 100x - 5xy, \quad \frac{dy}{dt} = -10y + 2xy.
   \]

   You should have just found in the last problem that \( x = 5, y = 20 \) is an equilibrium point and that the linearization oscillates about this equilibrium point (neither converging toward the equilibrium nor away from it). This is the uncertain case where the behavior of the linear system doesn’t determine the behavior of the non-linear system. Using the fact that \( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \), rewrite this autonomous system as a first order equation in \( x \) and \( y \) and then solve the equation to find an equation relating the populations \( x \) and \( y \). Based on this equation, do the populations converge to the equilibrium, away from the equilibrium, or oscillate about the equilibrium. Note that while this equation will tell you how \( x \) and \( y \) are related, it won’t tell you how they depend on the time \( t \).