Closed book. You may use a calculator and one 8 ½ × 11" sheet of handwritten notes (both sides). You must show your work to receive full credit. Write solutions in explicit form if possible. All problems have a solution that can be found using the techniques of this class. Problems that ask for answers in complete sentences will be graded on both content and clarity. Points may be deducted for errors in spelling and grammar.

**Pledge:**
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: _______________________________________      ___________

(signature)                                             (date)
1.
   a. What is the Laplace transform of \( f(t) = (t + 2)^2 \)?

   b. What is the inverse Laplace transform of \( F(s) = \frac{2s + 1}{s^2 + 3s - 18} \)?
2. Solve the initial value problem \( \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 13x = \delta(t) \), \( x(0) = 0 \), \( x'(0) = 0 \).
3. Solve the system

\[
\frac{dx}{dt} = -3x - 2y, \quad x(0) = 1
\]
\[
\frac{dy}{dt} = 9x + 3y, \quad y(0) = 2
\]
4. Solve the initial value problem
\[
\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 8x = f(t), \quad x(0) = 0, \quad x'(0) = 0.
\]
Write your answer as an integral involving the unspecified function \( f(t) \).
5. Match the following graphs of the poles of the Laplace transforms on the left with the graphs of the functions on the right.
6. Find and classify the equilibrium point of the system

\[
\frac{dx}{dt} = xy - 8,
\]

\[
\frac{dy}{dt} = y - x^2.
\]

For full credit, you must draw a sketch of the system in the \(xy\)-plane.
7. A mass of 10 grams is attached to an undamped spring with a spring constant of 10 g·cm/sec$^2$. The mass is at rest when it is struck by a hammer which delivers an impulse that we will represent as $\delta(t)$, which starts the spring moving. Exactly $\pi$ seconds later the mass is struck again by precisely the same impulse. Set up and solve the resulting differential equation and show the second impulse causes the mass to come to a complete stop.
8. Explain how and why we use a Dirac delta function to represent the force of a perfectly elastic collision. Your explanation should include

a. an explanation of why no ordinary function will work to represent the force (so we must use a “generalized” (idealized) function);

b. how we define the Dirac delta function in terms of integration;

c. a computation of the Laplace transform of the Dirac delta function; and

d. why you need to have a $u(t)$ term in the solution to problems like problem 2 on this exam.

Your explanation should be written in complete sentences and will be graded on both content and clarity (including spelling and grammar). You may use equations and/or refer to diagrams in your writing.
Name:____________________________________