Written Assignment #12:
Series Solution
Due 5:00pm Tuesday, May 4, 2004

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

1. If \( y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n \), then \( y(x_0) = a_0 \) and \( y'(x_0) = a_1 \). Find a formula for the \( k \)-th order derivative of \( y \) at \( x_0 \); i.e. find a formula for \( y^{(k)}(x_0) \) in terms of the \( a_n \)'s.

2. Find a power series solution \( y(x) = \sum_{n=0}^{\infty} a_n x^n \) for the following initial value problem:

\[
(1 - x^2)y'' - 6xy' - 4y = 0; \quad y(0) = 0, y'(0) = 1.
\]

Find a formula for \( a_n \) in terms of \( n \) (your formula should not be a recurrence relation) and give the interval of values of \( x \) for which the power series converges.