Written Assignment #3: Riccati Equations
Due 5:00pm Tuesday, February 17, 2003

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

1. Equations of the form \( \frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \) are called Riccati equations. If \( y_1(x) \) is a known particular solution to a Riccati equation, then the substitution \( v = y - y_1 \) can be used to transform the Riccati equation into a Bernoulli equation.

(a) If \( v(x) = y(x) - y_1(x) \), then what do \( y(x) \) and \( y'(x) \) equal (in terms of \( v \) and \( y_1 \))? 

(b) Suppose that \( y_1(x) \) is a particular solution to the Riccati equation

\[
\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x).
\]

Use the formulae found in (a) to make the change of variable \( v = y - y_1 \) and transform this equation into the form of a Bernoulli equation for \( v \).

2. In each of the following problems is a Riccati equation, a function \( y_1 \) and an initial condition. Verify that the function \( y_1 \) is a particular solution to the Riccati equation, then find a particular solution satisfying the given initial condition.

(a) \( y' = (y - x)^2 + 1; \quad y_1(x) = x; \quad y(0) = \frac{1}{2} \).

(b) \( y' = y^2 - \frac{y}{x} - \frac{1}{x^2}, \quad x > 0; \quad y_1(x) = \frac{1}{x}; \quad y(1) = 2. \)

(Note: you can make the change of variable \( v = y - y_1 \) to transform the Riccati equation to a Bernoulli equation and solve the resulting Bernoulli equation to obtain all solutions \( v = v(x) \). The solutions to the Riccati equation will then be given by \( y = v + y_1 \).)