Closed book. You may use a calculator and one 8 ½ x 11" sheet of handwritten notes (both sides). You must show your work to receive full credit. Write solutions in explicit form if possible. All problems have a solution that can be found using the techniques of this class. Problems that ask for answers in complete sentences will be graded on both content and clarity. Points may be deducted for errors in spelling and grammar.

Pledge:
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: ________________________________  ___________

   (signature)  (date)
1. Solve the initial value problem,
\[ y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0. \]
2. Find all solutions to $y'' + 8y' + 12y = \cos(2x)$. 
3. Solve the initial value problem
\[ y'' + 6y' + 9y = e^{-3x}, \quad y(0) = 1, \quad y'(0) = 0. \]
4. Match the following differential equations with the graphs of one of their solutions.

(a) \( y'' + 25y = 30\cos(5.5t) \)

(b) \( y'' + 25y = \cos(3t) \)

(c) \( y'' + y' + 25y = 30\cos(5t) \)

Equation: ________

Equation: ________

Equation: ________
5. Approximate $x(1)$ where $x'' + 4x' + \frac{t}{x} = t^2$, $x(0) = 1$, $x'(0) = 1$, using the improved Euler's method with a step-size of $h = 1$. Carry out all calculations to 4 decimal places.
6. Write $y(t) = \sin(3t) - \sqrt{3} \cos(3t)$ in the form $A \cos(\omega t + \varphi)$. What is the maximum value of $y(t)$?
7. Write a paragraph explaining the differences between overdamped and an underdamped springs. Your paragraph may include equations and may refer to graphs you draw. In grading the paragraph I will consider both content and clarity and will look for the following specific elements.

(a) What is the difference in behavior of overdamped and underdamped springs (this should be described in words as well as a graph)?

(b) What is the rule for classifying springs as underdamped or overdamped given the mass, damping, and spring constant?

(c) What is the mathematical justification for the rule in (b)?

This problem deals with material we may not have covered by the time of Exam 1.
8. Suppose a circuit has an inductor, a resistor, and a capacitor (so \( L, R, \) and \( C \) are all positive). The power for the circuit comes from a battery that supplies a constant 12 Volts. Show the steady-state current is 0.

This problem deals with material we won’t have covered by the time of Exam 1.
Name:____________________________________