Complex Variables and Linear Operators

1. Using Euler’s formula, $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and the usual rules of exponents we can establish De Moivre’s formula,

$$(\cos(n\theta) + i\sin(n\theta)) = e^{in\theta} = (e^{i\theta})^n = (\cos(\theta) + i\sin(\theta))^n.$$ 

Use this formula to write the following in terms of $\sin(\theta)$ and $\cos(\theta)$.

a. $\cos(6\theta)$

b. $\sin(6\theta)$

2. Show $Ly = Dy - x^2$ is not a linear operator. *Hint:* It is sufficient to show $L(y + z) \neq Ly + Lz$. 