All problems are worth 10 points. The exam is closed book, but you may use a calculator and one 8 ½ × 11" sheet of handwritten notes (both sides). You must show your work to receive full credit. Write solutions in explicit form if possible. Series solutions should be carried out at least to the $x^4$ term. All problems have a solution that can be found using the techniques of this class. Problems that ask for answers in paragraph form will be graded on both content and clarity. Points may be deducted for errors in spelling and grammar.

**Pledge:**
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: ________________________________  ___________

(signature)  

(date)
1. Find the general solution to \( \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 3y = \cos(2x) \).
2. Solve the initial value problem, \( \frac{dy}{dx} = y - xy^3, \quad y(0) = 2. \)
3. Solve the initial value problem
\[ y'' - xy' + (x - 2)y = 0, \quad y(0) = 1, \quad y'(0) = 2. \]
4. Solve the initial value problem

\[ \frac{d^2 x}{dt^2} + 4 \frac{dx}{dt} + 20x = \delta(t), \quad x(0) = 0, \quad x'(0) = 0. \]
5. Find the general solution to \( x^2 \frac{d^2 y}{dx^2} + 6x \frac{dy}{dx} + 6y = 0 \).
6. Find the \textit{a priori} lower bound for the radius of convergence of the series solution about $x_0 = 0$ for

$$(x^2 + 3x - 10)(x^2 + 2x + 2)y'' + (3x^3 - 24)y' + (2x + 7)y = 0$$
7. (a) Convert $3e^{-i\pi/4}$ to rectangular form.

(b) Convert $-\sqrt{3} - 3i$ to polar form.
8. A mass of 2kg is attached to a spring causing it to stretch .05m. The spring is then placed in free motion and the quasiperiod (the time for the spring to complete one cycle) is observed to be 1 second. What is the damping coefficient of the spring? Use $g = 9.8 \text{ m/sec}^2$. 
9. Find and classify the equilibrium point of the following system as moving directly out, directly in, spiral out, spiral in, or saddle.

\[
\frac{dx}{dt} = x - y, \\
\frac{dy}{dt} = \sin(x) + y.
\]

This problem covers material that won’t be on the Fall 05 final.
10. Verify that \( y(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \) solves the initial value problem,
\[ y'' - 2xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0. \]
11. Match the following equations with their slope fields.

(a) \( \frac{dy}{dx} = \cos(x) \) (A)

(b) \( \frac{dy}{dx} = \cos(y) \) (B)

(c) \( \frac{dy}{dx} = \cos(x + y) \) (C)
12. Write a paragraph or two that explains the importance of linearity. For full credit, you should include the following elements.
   (a) The definition of linearity.
   (b) An explanation of how linearity simplifies problems.
   (c) At least two different examples to illustrate your explanation (which may be other problems on this exam).
   (d) Finally, for contrast, list at least one non-linear problem (which may also be another problem on this exam).
Name:______________________________